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Simultaneous Search and Network Efficiency*

Pieter A. Gautier† and Christian L. Holzner‡

Abstract

When workers send applications to vacancies they create a network. Frictions arise because workers typically do not know where other workers apply to and firms do not know which candidates other firms consider. The first coordination friction affects network formation, while the second coordination friction affects network clearing. We show that those frictions and the wage mechanism are in general not independent. The wage mechanism determines both the distribution of networks that can arise and the number of matches on a given network. Equilibria that exhibit wage dispersion are inefficient in terms of network formation. Under complete recall (firms can go back and forth between all their candidates) only wage mechanisms that allow for ex post Bertrand competition generate the maximum matching on a realized network.

Keywords: Efficiency, network clearing, random bipartite network formation, simultaneous search.

JEL-Classifications: D83, D85, E24, J64

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1 Introduction

When workers apply to one or more jobs, a network arises where each application establishes a link between a worker and a firm. In such a decentralized environment there are two coordination frictions, (i) workers do not know where other workers apply to and (ii) firms do not know which workers are considered by other firms. We can think of the first coordination friction as referring to random network formation, while the second coordination friction affects network clearing (the number of matches on a given network). Treating the search process as a matching on a bipartite network gives new insights into one of the key questions in the labor-search literature namely, under which conditions is the decentralized market outcome constraint efficient? With constraint efficiency we mean that the market outcome is identical to the outcome of a hypothetical social planner who maximizes social welfare given the fundamental frictions. The main contribution of our paper is that it shows how under directed search (workers observe the wage before applying to a job), the wage mechanism affects frictions through network formation and clearing.[1]

We find that efficient network formation requires that all vacancies should receive an application with the same probability and that efficient network clearing requires ex post Bertrand competition between firms that consider the same candidate. Random search, where each vacancy has the same contact probability and directed search without ex-ante wage dispersion, lead to efficient random network formation. The efficiency condition in Kircher (2009), where workers send multiple applications and firms can contact all workers, dictates however that some vacancies should have a higher probability to receive an application than others. The difference between our efficiency condition and Kircher's occurs because he places more restrictions on the planner's network clearing mechanism.

Wage mechanisms that allow for ex post Bertrand competition are socially efficient in terms of network clearing, because they generate the maximum number of matches

[1]Coles and Eeckhout (2003) and Eeckhout and Kircher (2010) show that the number of matches in a model with identical workers is independent of the posted wage mechanism. We show that this does not occurs if workers send multiple applications. In the random search models of Diamond (1982), Mortensen (1982) and Pissarides (2000) the wage determination process and the matching process are fully independent. In Moen's (2000) competitive search model, workers can sort in sub markets which are characterized by different wage and market tightness pairs. Within each sub market, given market tightness, the number of matches does not depend on wages. When workers apply to only one job, only the first coordination friction occurs, since all firms that receive at least one application can be sure that their selected candidate has no competing offer from another firm, see Burdett, Shi and Wright (2001).

possible. This happens because firms can increase their wages in subgraphs with an excess number of vacancies. Firms in subgraphs with an excess number of workers do not have to increase their posted wages. Ex post Bertrand competition therefore solves the second (between-firm-coordination) friction.

To the best of our knowledge we are the first to analyze how decentralized wage mechanisms affect network clearing in a decentralized search model with complete recall where workers only know where they send their own applications and firms only know which workers applied to them. Part of the network literature has analyzed different pricing mechanisms and has studied whether these price mechanisms lead to an efficient matching of sellers and buyers. Kranton and Minehart (2001) show for example that a public ascending price auction ensures efficient network clearing. Corominas-Bosch (2004) shows for identical sellers and buyers that an alternating offers game where all sellers (or buyers) of a subgraph simultaneously announce prices leads to a maximum matching. This literature, however, assumes that once a network has been formed all agents know the complete network (or the entire subgraph of the network they are in).[2] This knowledge allows sellers and buyers to determine their outside option trading partners and trading prices. We show that ex post Bertrand competition achieves the maximum matching, even if agents do not know the network structure. Another part of the network literature uses the set-valued approach, i.e., it either starts with a set of competitive price vectors and shows that the resulting matches are pairwise stable and maximize aggregate welfare (see Kranton and Minehart, 2000), or it starts by assuming that pairwise stable matches must arise and then analyses the entry decision of agents (see Elliott, 2011b). Those papers do not layout the game that leads to a competitive price vector or a pairwise stable matching like we do. Finally, there is a growing number of papers that combine insights from search and network theory.[3] Those papers focus mainly on how social networks of workers can pass information of the location of jobs on to each other which is different from the bipartite network (between workers and firms) framework in our paper.

Complete recall is essential to achieve efficient network clearing. If firms can select at most one candidate to which they are linked, the resulting equilibrium is typically not constraint efficient, since no price mechanism can resolve the coordination friction between firms. In the search literature, Albrecht, Gautier and Vroman (2006) and Galeanos and

[2] Galeotti et al. (2010) analyse network games with limited information. However, they only consider one type of agents, i.e., they do not consider vacancies and workers or sellers and buyers in a bipartite network.

[3] Example include, Boorman (1975), Calvó-Armengol and Jackson (2004), Calvó-Armengol and Zenou (2004), Fontaine (2004).

Kircher (2009) consider a framework with limited recall that leads to inefficient network clearing.[4] In the network literature, Manea (2011) considers a network game that is similar to a limited recall environment. He analyzes a framework, where agents that are connected in a network are randomly selected to bargain. During the bargaining game they are not able to contact other connected agents. We also show that complete recall by itself like in Kircher (2009), where firms commit to their posted wages, does not lead to the maximum number of matches.

Although a search environment without wage dispersion and with ex post competition leads to efficient network formation and network clearing, it may still be inefficient in other dimensions that we ignore here (i.e. search intensity and vacancy creation). Kircher (2009) shows for example that efficient entry and search intensity requires wage dispersion and commitment. Combining our and his results, suggests that there may not exist a decentralized mechanism that is efficient in all dimensions.

The paper is organized as follows. We start in section 2 with a 3-by-3 example that illustrates our main point that wage dispersion leads to less efficient networks and ex post Bertrand competition generates a maximum matching on a given network while wage commitment does not. Sections 3 and 4 consider a large labor market. In section 3 we describe the timing of events and the network formation and clearing process. In section 4 we apply some basic insights from graph theory to derive two important general results. First, in section 4.1 we show that ex-post Bertrand competition with complete recall gives the maximum matching on a given network and wage mechanisms without ex-post competition do not. In section 4.3 we show that in terms of network formation, workers should apply to each vacancy with equal probability. This only occurs, if all firms post the same wage or if search is random (workers do not observe the wage ex ante). Finally section 5 concludes.

2 An example

This section illustrates our main points that (i) ex ante wage dispersion leads to less efficient network formation and that (ii) ex post Bertrand competition generates a maximum matching. We consider the following two dimensions corresponding to random network formation and network clearing under incomplete information, (1) random search versus directed search (note that random search implies that each vacancy receives an application with the same probability), and (2) ex post Bertrand competition versus wage

[4] Albrecht et al. (2006) also allow ex post Bertrand competition.

commitment.

Consider a simple economy with 3 unemployed workers and 3 firms, each with one vacancy, ($u = v = 3$) and where workers send two applications ($a = 2$).[5] First, we look at network formation and assume that network clearing generates the maximum number of matches. Then, we look at which wage mechanisms are most efficient in terms of network clearing. Efficient network clearing implies that the number of matches is equal to 3, if each of the three vacancies receives at least one application, and equal to 2, if only two vacancies receive applications. Note that these are the only two possible outcomes, since no worker sends both applications to the same firm. Let ξ_i be the probability that a worker sends one of her two applications to vacancy i. The maximum number of matches is,

$$M = \sum_{i=1}^{3} \left(1 - (1 - \xi_i)^3\right), \text{ with } \sum_{i=1}^{3} \xi_i = 2,$$

where $(1 - \xi_i)^3$ equals the probability that vacancy i does not get any application. Since the function $\left(1 - (1 - \xi_i)^3\right)$ is concave in ξ_i, Jensen's inequality implies that the number of matches is maximized, if all vacancies have the same probability to receive an application, i.e., if $\xi_i = 2/3$. Thus, only wage mechanisms that generate no ex ante wage dispersion (which is always the case under random search) can lead to the maximum number of matches, $M = 26/9 \approx 2.889$. In Appendix A we consider four cases that depend on the search environment (random or directed search) and on the firm's strategy space (i.e., can firms increase their initial offers or not). In all cases we allow for complete recall (firms can go back and forth between their candidates) and fully characterize equilibrium wages and the expected matching rates.[6] It turns out that in this example, there is always wage dispersion under directed search. So, interestingly, random search is most efficient in terms of network formation. In the case of directed search with ex-post competition, the amount of wage dispersion is a lot smaller than in the case of commitment.[7] In

[5] If workers send 1 application or 3 applications, the number of matches generated is independent of the wage mechanism used.

[6] With the exception of Kircher (2009), who studies directed search with wage commitment, all those cases have been studied with limited recall. For directed search with ex-post Bertrand competition, see Albrecht et al. (2006), for random search with ex post competition see Gautier and Wolthoff (2009), for directed search with commitment and no ex post competition see Galeanos and Kircher (2009) and for random search with commitment, see Gautier and Moraga Gonzalez (2004) (all those papers have no complete recall except the last one, which considers complete recall in a 3by3 example).

[7] We conjecture that in a large market, the wage dipersion will completely disappear in the case with ex post competition.

this ex post competition case, only an equilibrium exists with one high wage and two low wage firms. The high wage firm has an application probability of $\xi_h \approx 0.722$ and the low wage firms a probability of $\xi_l \approx 0.639$ (the equilibrium is fully characterized in Appendix A.3). The total number of matches is given by $M \approx 2.884$. Under directed search and wage commitment, there is more wage dispersion; the high wage firm has an application probability of $\xi_h \approx 0.956$ and the low wage firms of $\xi_l \approx 0.522$ (details are in A.4). As we will show below, in the case of directed search and wage commitment, both network formation and network clearing is inefficient. To isolate the effect of the wage mechanism on network formation we calculated the total number of matches, imposing efficient network clearing (which in general does not occur in equilibrium). In that case, $M \approx 2.781$. Summing up, directed search with ex post competition generates more efficient networks than without ex post competition, because the latter case has more wage dispersion.

Next, consider network clearing. Efficient network clearing requires that the number of matches is equal to 3, if all three vacancies are collectively linked to all three workers, and that the number of matches is equal to 2, if only two vacancies are collectively linked to all three workers. The later is always ensured, since both vacancies with applications received three applications and are linked to all three workers. To see why ex post Bertrand competition leads to 3 matches, if three vacancies are collectively linked to three workers, we show that one gets a contradiction if this does not hold. Suppose a worker and a firm remain unmatched in this case. This implies that the unmatched worker receives her reservation value. The firm that is linked to the unmatched worker must pay a wage equal to the reservation value to its matched worker, since any higher wage would not be profit maximizing. The unmatched firm, however, is willing to pay a wage equal to the marginal product. Thus, the worker who is linked to the unmatched firm but hired by another firm must be paid a wage equal to his marginal product, since any lower wage would be outbid by the unmatched firm. Thus, one of the three firms pays the reservation wage, one the marginal product and one remains unmatched. The unmatched worker cannot be linked directly to the unmatched firm, since both parties would then form a match. Thus, the unmatched worker can only be linked to both matched firms. This, however, implies that both matched firms must pay a wage equal to the reservation value. This cannot be the case as we argued above. Thus, ex post Bertrand competition leads to the maximum number of matches possible.

Network clearing is in general not efficient, if firms commit to their posted wages. To see this, consider the graph in Figure 1, which pictures a particular realization of the

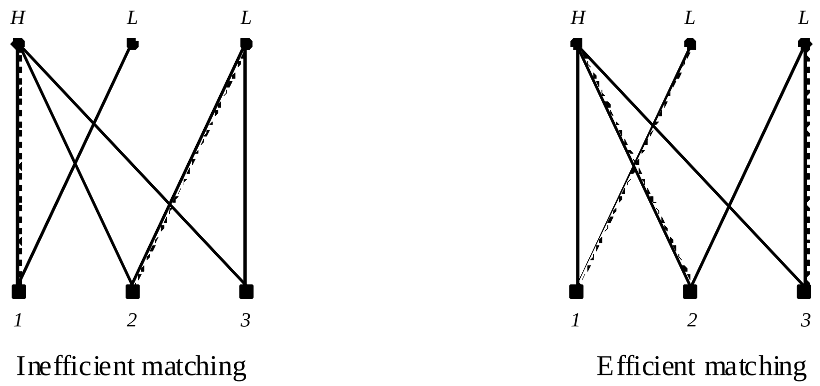

Figure 1: Inefficiency without ex post competition

case where each worker sends one application to the high-wage firm and one to one of the two low-wage firms (thick lines). The number of matches (dashed lines) now depends on which worker is chosen by the high-wage firm. If the high-wage firm offers the job to one of the workers who are linked to the low-wage firm with two applicants, i.e., to worker 2 or 3 in Figure 1, the number of matches is equal to the maximum number of matches (3). If the high-wage firm offers the job to the worker linked to the low-wage firm with only one applicant, i.e. to worker 1 in Figure 1, there will be only two matches, since the low-wage firm with only one applicant will remain unmatched. The expected number of matches in a model with directed search and wage commitment is therefore lower (in Appendix A.4 we derive the equilibrium wages and show that $M \approx 2,538$). So without ex post competition, the number of matches can even be inefficient if all firms post the same wage. In this case there also exists a positive probability that the worker that is linked to the firm with only one applicant is hired by the firm with three applicants.

Network clearing is also not efficient under random search with wage commitment. Gautier and Moraga Gonzalez (2004) study such an environment and give a 3 by 3 example, which we just summarize here. For the same reasons as in Burdett and Judd (1983) and Burdett and Mortensen (1998) no symmetric-pure strategy equilibrium exists and wages are offered from a continuous distribution. The equilibrium wage distribution is determined by the equal profit condition and the fact that **the** lowest wage offer equals the reservation value. The total number of matches is equal to $M = 73/27 \approx 2,703$.

The following table summarizes the expected number of matches that are realized in equilibrium for the different search environments and wage mechanisms.

This illustrates that the wage mechanism and the matching process are not inde-

	random search	directed search
ex post competition	$2,889$	$2,884$
wage commitment	$2,703$	$2,538$

Table 1: Expected number of matches under different search and wage mechanisms

pendent. Different search environments generate different distributions of networks and whether the wage mechanism allows for ex post competition or not affects the number of trades on a given network.

3 Framework

Before presenting our main results for a large labor market, we first lay out the precize setting and the timing of events. Consider v identical firms with one vacancy each and u identical risk neutral unemployed workers, who can send $a \leq v$ applications to different firms. Workers have a reservation wage of 0 and a matched firm-worker pair produces 1. As is standard in the directed search literature we impose both symmetry and anonymity. Symmetry implies that identical workers play identical strategies while anonymity implies that firms must treat identical workers similarly and vice versa (see Burdett, Shi and Wright, 2001). In our directed search framework we allow firms to post a wage with the possibility to Bertrand compete ex-post. Then, we compare our results to Kircher (2009) where firms post fixed wages and cannot Bertrand compete ex-post. Random search models can be analyzed in this framework by assuming that all firms post the reservation wage (which is 0 here) in the first stage.

The timing is as follows:

1. Firms post a wage $\underline{w}$. The actual wage w paid by the firm can be higher than the posted wage, if firms can (Bertrand) compete for their candidates with other firms that are also connected to this worker in later stages of the game.

2. Workers send out $a \geq 2$ applications.

3. Each firm selects a worker (if present) and offers the worker its posted wage $w = \underline{w}$ from stage 1. The offers are verifiable.

4. If a worker gets one offer w, she informs all firms where she applied, except the one that made the offer, that she will only be willing to work for a wage $w' =$

$w + \epsilon$ or higher. If a worker has multiple offers $\{w^1, w^2, ..., w^j\}$, she informs all firms, where she applied, except the one that offered the highest wage ($w^h = \arg\max\{w^1, w^2, ..., w^j\}$), that she will only be willing to work for a wage $w' = w^h + \epsilon$ or higher.

5. If the worker that the firm selected did not ask for a higher wage, the firm offers the same wage w again. If the worker that the firm selected asks for a higher wage $w' > w$, the firm offers one of the candidate(s) that did not ask for a wage higher than w the job at the posted wage w. If there is no candidate with a request $w' \leq w$, the firm picks the worker with the lowest request $w^l = \arg\min\{w^1, w^2, ..., w^j\}$ and offers her the job at the wage w^l, as long as the wage does not exceed the marginal product, i.e., $w^l \leq 1$.

6. If at least one worker received a higher offer than in the previous stage, the game goes back to stage 4. If all workers received the same wage offer as in the previous stage, matches are formed. A firm fails to hire, if it has no applicants or if all its candidates choose other firms. A worker remains unemployed, if she received no offers.

Note that workers and firms do not observe the network. Firms only know how many workers applied to them and whether a worker is willing to work for the offered wage. The ability to go back and forth between workers constitutes a small but important difference to the Bertrand game proposed by Albrecht, Gautier and Vroman (2006), where firms can create a shortlist of two workers and cannot go back to a worker once they decided to contact the next worker on the shortlist. The ability to go back and forth between workers is, however, crucial to achieve efficient network clearing.

In terms of network formation, our framework is similar to Albrecht et al. (2004). This differs from the standard random network formation process of Erdös-Renyi where (in a labor market context) an application is sent to a particular firm with probability p.[8] In our setting, all workers send a applications.[9]

Firms find it optimal to follow the strategies laid down in the Bertrand game above, since they only have to increase their wage offer, if none of their candidates is willing to work for the wage offered. Workers' behavior in the proposed Bertrand game is also optimal. They prefer to communicate that they have one or more offers to the firms

[8] See Bollobas (2001) for a bipartite version.

[9] If there is no wage dispersion and the market is large, each application is sent with probability $1/v$ to each firm.

that did not respond in order to engage them into Bertrand competition. Furthermore, since workers do not ask the firm that offered the highest wage to increase its wage offer, workers make sure that these firms will not contact another worker and that they are at least able to work for the highest wage offered so far.

We take the number of applications that workers send out and market tightness as given. The main reason for this is that the conditions for efficient entry and the number of applications are well known and have been studied before.[10] This allows us to focus on the efficiency of random network formation and clearing. It is however important to keep in mind that a wage mechanism that generates efficient networks may not be efficient in terms of market tightness or the number of applications and vice versa.

4 General results on random network formation and network clearing

The example of section 2 suggests that ex ante wage dispersion is inefficient in terms of random network creation and that we need Bertrand competition in order to get efficient network clearing. In this section we use some results from graph theory to show that those results hold in more general settings. In section 4.1 we show that maximum matching requires ex post competition and in section 4.3 we show that it is desirable from a social point of view that the application arrival rate is the same for all vacancies.

4.1 Maximum matching requires ex-post competition

In this section we show that for a given network, ex-post Bertrand competition with complete recall generates a maximum matching. The network clearing mechanism that is necessary to achieve a maximum matching also implies that committing ex-ante to a specific wage without allowing for ex-post Bertrand competition does typically not generate the maximum matching. Below, we first briefly describe some basic concepts of graph theory that are relevant for our environment.

[10] Gautier and Moraga-Gonzalez (2005) and Albrecht et al. (2006) find without recall, that workers send too many applications (due to rent seeking and congestion externalities) and that entry is excessive, because firms have too much market power. Kircher (2009) shows that with directed search, wage commitment and full recall, entry and search intensity are socially efficient. Elliot (2011b) finds efficient entry but workers send too many applications.

When workers apply to jobs, each of their applications is a link (or edge) in a bipartite network. The wage mechanism and search environment determine both the distribution of networks that can arise and the matching on a given network. In our environment, a typical network realization consists of several disjoint graphs. Each worker and firm is a node (or vertex). The graphs in our environment are simple (workers do not send multiple applications to the same firm), undirected (if worker i is linked to firm j, then firm j is linked to worker i) and bipartite ($G = \langle u \cup v, L \rangle$ consists of a set of nodes formed by two different kind of agents, i.e., by workers $\{u_1, ...u_n\}$ and vacancies, $\{v_1, ...v_m\}$, and a set of links L where each link connects a worker to a firm so workers are not linked to other workers and firms are not linked to other firms).

Definition 1: *A matching M in a graph G is a set of links such that every node of G is in at most one link of M.*

Central to our result that a maximum matching requires ex-post competition is the following theorem by Berge,

Berge's Theorem (1957):
A matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path.

In our bipartite graph environment an M-augmenting path is defined as a path where

1. worker-firm links that are part of the matching M alternate with worker-firm links that are not part of the matching M (definition of an M-alternating path) and

2. neither the origin (firm or worker) nor the terminus (worker or firm) of the path is part of the matching M.

Figure 2 depicts an M-alternating path and an M-augmenting path in a particular network. The dots represent vacancies and the squares unemployed workers. The solid lines represent applications ($a = 2$) and the dashed lines represent matched worker-firm pairs. The M-alternating path ($A - 1 - B - 2 - C - 4$) starts with the matched vacancy A and ends at the matched worker 4. The M-augmenting path ($A - 1 - B - 2 - C - 4$) in the second panel of Figure 2 starts with an unmatched vacancy, A, and ends with an unmatched worker, 4.

Berge's Theorem, translated to our setting, implies that a maximum matching in a graph is only guaranteed, if an unmatched firm is not linked to an unmatched worker via an M-augmenting path. The reason that a matching is not optimal, if an M-augmenting

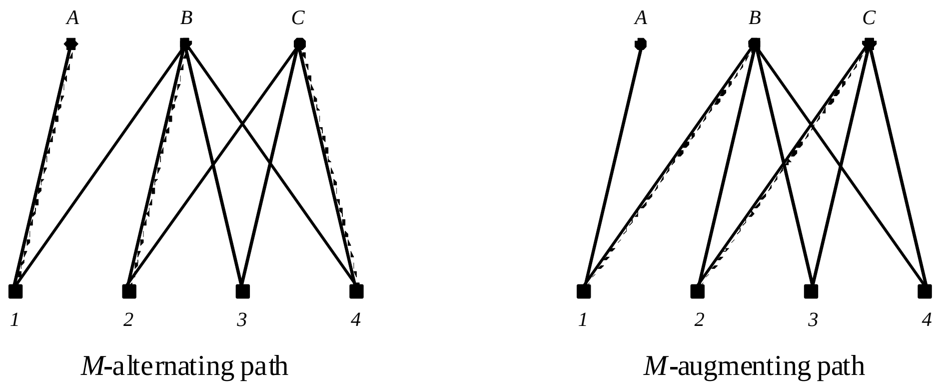

Figure 2: M-alternating path and M-augmenting path

path exists is that one could create one more match by switching the links. Then, the unmatched firm at the start of the M-augmenting path and the unmatched worker at the end of the M-augmenting path will both be matched and all worker-firm pairs that were matched before are rematched with another partner. Comparing the two paths in the second panel of Figure 2 illustrates this. The matching $M = \{1 - B, 2 - C\}$ in an M-augmenting path can always be increased by *switching* the dashed and solid links resulting in an extra link, i.e., $M = \{A - 1, B - 2, C - 4\}$.

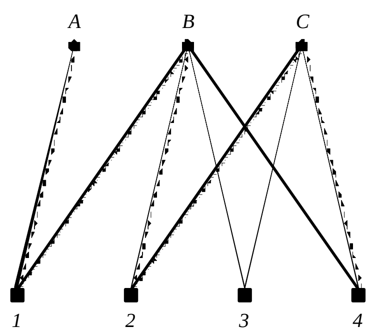

Figure 3: The maximum matching has no M-augmenting path

What remains to be shown is that if a matching M has no M-augmenting paths it is a maximum matching. This can be proven by contradiction. Suppose that in a particular graph in our setting there is a matching N ($A-1, B-2, C-4$; i.e., dashed lines in Figure

3) with more links than M $(1-B, 2-C$; i.e., dotted lines in Figure 3) , i.e., $|N| > |M|$. Then consider the symmetric difference $N\Delta M$ defined as the set of links that is either in N or M but not in both (the sum of dashed and dotted lines in Figure 3). Each worker or firm can have at most 2 links in $N\Delta M$ because it is hired by at most one firm in M and at most one firm in N. Since by assumption N is strictly bigger than M there must be at least one path in $N\Delta M$ with an odd number of links that starts with a firm (worker) in N and ends with a worker (firm) in N (i.e. $A-1-B-2-C-4$). But then this is an M-augmenting path because the firm and worker at the start and end of the path are (by the symmetric difference operation) not in M.

Thus, in order to show that Bertrand competition leads to a maximum matching we need to rule out that an M-augmenting path exists. In order to do so, we start with some properties resulting from the ex-post Bertrand competition game in section 3.

Lemma 1 *The highest posted wage is strictly smaller than* 1*.*

Proof: Under directed search, any firm that offers the highest wage and sets it equal to 1 makes no profit and could increase its profits by offering a wage strictly less than one since there is a positive probability that one of its candidates receives no better offers and accepts. If $a = v$, all firms know that they will hire a worker for sure. Since firms make take-it-or-leave-it wage offers, it is optimal for them to always offer the workers' reservation wage. We can think of random search as the case where all posted wages are zero. ■

Lemma 2 *If a worker remains unmatched, each firm along the M-alternating path that starts with the unmatched worker pays no more than the highest posted minimum wage.*

Proof. First, note that the unmatched worker cannot be linked to a firm with no other candidates, since that firm would hire the worker. Next, suppose that the unmatched worker applied to at least one firm with more than one other candidate. Since the worker remains unmatched, any firm where the worker applied to must pay its matched worker the wage it posted (under random search, it will pay the reservation wage). Otherwise, it could offer the unmatched worker its posted wage and the worker would accept this offer given that his reservation wage equals zero. Suppose now that contrary to Lemma 2, one of the firms along an M-alternating path (firm B), pays more than the highest posted wage, i.e. $w^* > \underline{w}^h$. We will show that this violates profit maximization. Since any of the other firms that is linked to the unmatched worker pays its posted wage to its matched worker, there exists an M-alternating path that starts at the unmatched worker and includes at least one firm that pays its posted wage and firm B that pays $w^* > \underline{w}^h$. But then there exists a firm (possibly firm B) along the M-alternating path that pays a wage $w^* > w^h$, but has an applicant that earns w^h or less. This firm could make higher profits, if it would offer the other applicant the job at a wage $\underline{w}^h + \varepsilon$ (note that each firm on an M-alternating path must have another candidate). ■

Lemma 3 *If a firm remains unmatched, then all workers along the M-alternating path that starts with the unmatched firms must earn a wage equal to the marginal product, i.e., $w = 1$.*

Proof. If a firm with candidates (firm A) remains unmatched, then its applicants must earn a wage $w = 1$, since at any wage $w < 1$, the firm could attract an applicant and make positive profits. Suppose there exists a firm, call it firm B, that pays a wage $w^* < 1$ to its matched worker. Then, there exists at least one firm along the M-alternating path that starts at the unmatched firm A and includes firm B that pays a wage $w = 1$, while the worker who is hired at B earns $w^* < 1$. But then this firm that pays a wage $w = 1$ could make higher profits, if it would offer one of its other candidates (again, each firm on an M-alternating path must have another candidate) the job at the wage $w^* + \varepsilon$. Thus, if one firm along an M-alternating path pays a wage equal to the marginal product, all firms along the M-alternating path must do so as well. ■

According to Berge's Theorem a maximum matching exists if and only if there is no M-alternating path that starts with an unmatched worker and ends with an unmatched firm, i.e., if and only if there is no M-augmented path. Given the wage pattern in an M-alternating path that starts with an unmatched worker (Lemma 2) or with an unmatched firm (Lemma 3), we can write down our main Theorem.

Theorem 1: *Ex-post Bertrand competition leads to a maximum matching in all graphs of the network.*

Proof: Suppose it would not lead to a maximum matching. In that case there would exist an M-augmenting path with at least one unmatched worker and one unmatched firm. But then Lemma 1,2 and 3 imply that all firms along the M-augmenting path (that is also an M-alternating path) offer both a wage less than 1 and a wage equal to 1, which is a contradiction. ■

The flexibility to adjust wages ex-post is central to achieve efficiency in network clearing. If firms commit to their posted wages and do not adjust their wages ex-post, we can typically observe different wages along an M-alternating path. If both end nodes of the M-alternating path are unmatched, i.e., if we have an M-augmenting path, there is no mechanism inherent in the matching process associated with wage commitment that can induce the matched firm-worker pairs to rematch with the unmatched firm and worker at the end of the M-augmenting path. Thus, if firms commit not to increase their posted wages ex-post, network clearing is generally not efficient. Note, that the 3 by 3 model of section 2 also gives an example where network clearing is not efficient due to the lack of Bertrand competition. Thus, Berge's Theorem also implies the following Corollary:

Corollary 1: *If firms commit not to increase their posted wages ex-post, network clearing is typically inefficient and the maximum matching is not realized.*

Corollary 1 shows that directed search models with fixed posted wages are not able to solve the second coordination friction (firms do not know which workers are considered by other firms). Thus, although directed search with fixed posted wages is constraint efficient in terms of firm entry and number of applications that workers send, see Kircher (2009), it generally does not generate the maximum matching that is possible given the network that is formed between firms and their applicants.

Theorem 1 also implies that a social planner would never want to give one subgroup of firms the right to match first regardless of the network. Such a property arises, if some firms offer higher wages than others and wages cannot be raised ex-post as in Kircher (2009).

Corollary 2: *It is socially inefficient to have a subgroup of firms that matches first.*

Corollary 2 implies that it is socially inefficient to have a subgroup of high wage firms that match first and a subgroup of low wage firms that match only if their candidate(s) receive no offers at a high wage firm.[11]

4.2 Wages

Lemmas 1 to 3 are also informative about the payoffs that workers and firms receive. According to Lemma 3 all workers that are part of an M-alternating path that includes an unmatched firm earn a wage equal to the marginal product, i.e., $w = 1$, if firms can ex post Bertrand compete for their candidates. Lemmas 1 to 3 also imply that all workers

[11] Note, that Kircher's (2009) equilibrium is constrained efficient because the planner takes the existence of a subset of firms that match first as given, whereas here this is not part of the planner's constraint.

that are part of such an M-alternating path, must be matched and earn a wage equal to their marginal product. Thus, these M-alternating paths are characterized by an excess number of firms. Similarly, there are M-alternating paths that are characterized by an excess number of workers. According to Lemma 2 all workers that are part of such an M-alternating path earn a wage no higher than the highest posted wage. Lemmas 1 to 3 also imply that all firms that are part of such an M-alternating path must be matched. Lemmas 1 to 3 also allow for M-alternating paths with equal number of workers and firms where all workers and firms are matched. In order to determine the wages paid in such even subgraphs we use the properties of the Decomposition Theorem by Corominas-Bosch (2004), which – in terms of our terminology – decomposes a network into firm-, worker- and even subgraphs. A firm subgraph contains more firms than workers and workers are paid their marginal product. A worker subgraph contains more workers than firms and workers are paid a wage no higher than the highest posted wage. In even subgraphs the number of workers equals the number of firms.

Decomposition Theorem (Corominas-Bosch, 2004):

(1) *Every graph G can be decomposed into a number of firm subgraphs (G_1^f,..., $G_{n_f}^f$), worker subgraphs (G_1^w,..., $G_{n_w}^w$) and even subgraphs (G_1^e,..., $G_{n_e}^e$) in such a way that each node (firm or worker) belongs to one and only one subgraph and any firm (worker) in a firm-(worker-)subgraph $G_{i,}^f$ ($G_{i,}^w$) is only linked to workers (firms) in a firm-(worker-)subgraph G_j^f (G_j^w).*

(2) *Moreover, a given node (firm or worker) always belongs to the same type of subgraph for any such decomposition. We will write $G = G_1^f \cup ... \cup G_{n_f}^f \cup G_1^w \cup ... \cup G_{n_w}^w \cup G_1^e \cup ... \cup G_{n_e}^e$, with the union being disjoint.*

Such a decomposition into firm-, worker- and even subgraphs plus some extra links can be obtained by following an algorithm introduced by Corominas-Bosch (2004), see step 2 in Appendix C for the exact algorithm. The algorithm first looks for firm subgraphs and separates all of them from the network. Then it identifies worker subgraphs and removes all of them from the network. The remaining subgraphs are even subgraphs. The decomposition is not unique, since the exact splitting of nodes (firms or workers) into subgraphs can differ, because the algorithm uses the subindex of a node in order to start finding the subgraphs. The second statement of the Decomposition Theorem, however, states that any firm and any worker will always belong to the same type of subgraph, a property important to guarantee that the different possible decompositions are payoff equivalent.

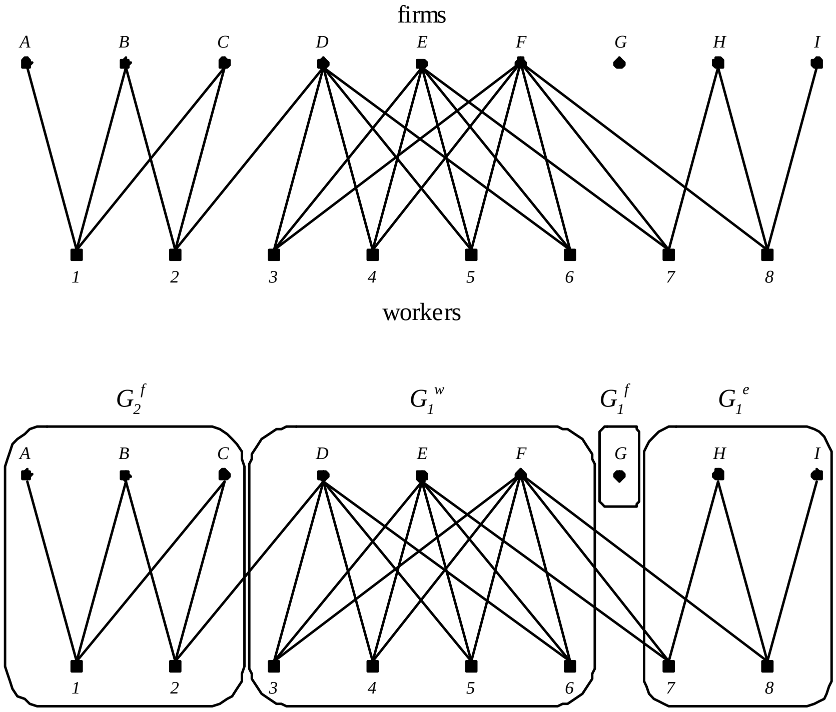

Figure 4: Graph-Decomposition

Figure 4 illustrates the Decomposition Theorem. The algorithm starts with the first firm and identifies a set of firms as firm subgraph if it has less neighbors (more precisely, if it is jointly linked to less neighbors, i.e., $|F| < |N(F)|$). In order to ensure that the maximum matching is found, the algorithm has to start with $|F| = 1$. The number $|F|$ increases by one once, all firm combinations with $|F|$ have been considered (Hall's Theorem, 1935). The first subgraph in Figure 4 is the unmatched firm G. The firm subgraph G_1^f is removed before the algorithm continues. Since there are no firm subgraphs with $|F| = 2$, the next firm subgraph has three firms, i.e., $|F| = 3$, The three firms A, B and C in this subgraph are collectively linked to workers 1 and 2, i.e., $N(\{A, B, C\}) = \{1, 2\}$ and $|N(\{A, B, C\})| = 2$. Once the firm-subgraph G_2^f is removed, it is easy to identify that the remaining sets of firms are collectively linked to more neighbors, i.e., $|F| \geq |N(F)|$. Hence, there are no further firm subgraphs. The algorithm continues by looking for worker subgraphs in the same way as it looked for firm subgraphs. At $|W| = 4$, the algorithm identifies a worker subgraph with $N(\{3, 4, 5, 6\}) = \{D, E, F\}$ and $|N(\{3, 4, 5, 6\})| = 3$. Once the worker subgraph G_1^w is removed, and no further worker subgraph are found the algorithm stops by identifying all remaining subgraphs as even subgraphs, i.e., in Figure 4 the remaining subgraph G_1^e is an even subgraph with $N(\{7,8\}) = \{H, I\}$ and $|N(\{7,8\})| = 2 = |\{H, I\}|$.

The decomposition theorem of Corominas-Bosch (2004) is also useful for the analysis of network formation that we discuss in the next section, because it allows us to determine which kind of links formed by an additional application will result in an extra match. Since all firms in even subgraphs and worker subgraphs are matched, only applications from workers in worker subgraphs (which includes unmatched workers) to firms in firm subgraphs (which include firms without any application) will result in additional matches.

An alternative way to interpret the decomposition is in terms of splitting firms and workers into "strong", "weak" and "even" firms and workers depending on their capability to extract the maximum surplus from their matched partners (see Corominas-Bosch, 2004, p. 51). Workers in firm subgraphs are "strong" nodes, since they earn a wage equal to their marginal product. Similarly, firms in worker subgraphs are "strong" nodes, since they are able to extract the maximum surplus conditional on the posted wage. Contrary, workers in worker subgraphs and firms in firm subgraphs are "weak" nodes and workers and firms in even subgraphs are even nodes. The first part of the Decomposition Theorem states that a worker (firm) in a worker-(firm-)subgraph can only be connected to firms (worker) in other worker-(firm-)subgraphs, which implies that "weak" nodes can only be linked to "strong" nodes. This also implies that "even" nodes cannot be linked to "weak" nodes, or in terms of our model, that firms in even subgraphs cannot be linked to workers in worker subgraphs or that workers in even subgraphs cannot be linked to firms in firm subgraphs. Thus, the outside option of workers in even subgraphs is at most the highest posted wage, since they can only be linked to firms in even subgraphs or worker subgraphs. This last property is important to determine the wages in even subgraphs.

Lemma 4 *(i) Firms in firm subgraphs pay a wage equal to the marginal product.*
(ii) Firms in worker subgraphs pay a wage no higher than the highest posted wage.
(iii) Firms in even subgraphs pay a wage no higher than the highest posted wage.

Proof: (i) and (ii) follow immediately from Lemmas 1 to 3. To prove (iii) consider the following properties of an even subgraph. In an even subgraph that results from the decomposition algorithm introduced by Corominas-Bosch (2004), workers are either linked to firms in even or in worker subgraphs. Part (ii) of the Lemma implies that the wage offers made by firms in worker subgraphs to workers in even subgraphs are no higher than the highest posted wage. To establish part (iii) it remains to be shown that firms in even subgraphs never have an incentive to offer a wage above the highest posted wage given the Bertrand game outlined in section 3. A firm only increases its wage offer above the highest posted wage, if all workers that are linked to it ask for a wage above the highest posted wage. According to Step 4 of the Bertrand game a worker only asks for a wage above the highest posted wage, if at least one of the firms, where the worker applied to, offers him a wage above the highest posted wage. The first firm that offers a wage above the highest posted wage cannot be part of the even subgraph. If it were part of the even subgraph, then another firm that is also part of the even subgraph must have offered a wage above the highest posted wage before. Thus, no firm in an even subgraph can be the first to offer a wage above the highest posted wage. ■

Directed search with ex post competition generates ex post wage dispersion similar to Albrecht, Gautier and Vroman (2006). The knowledge about wages paid in the different subgraphs allows us also to gain some insight into the payoffs that firms get in different subgraphs. This will be useful for analyzing efficiency in network formation.

4.3 Efficient network formation

In our setting, network formation is random. The symmetry and anonymity assumptions do not allow workers to identify certain firms and to condition their application decision on firms' names. The limited information available to workers leads to random network formation.[12] Workers might, however, know certain characteristics of firms, for example the posted wage, and condition their application decision on those observed characteristics.

We consider an urn-ball model of network formation, (see Albrecht, Gautier and Vroman, 2004) where workers randomly send out a applications to different firms.[13] Each application can be thought of as creating a link in a bipartite graph. This process differs from the seminal Erdös and Rényi (1960) random network formation model where each link is formed with a certain probability and the number of applications that a worker sends is a random variable. In our framework the number of applications that each worker sends is given and the randomness comes from the fact that workers do not know where other workers apply. The number of applications that a firm receives is therefore a random variable. Under directed search, the expected number of applications a firm receives

[12] Network formation is deterministic, if workers decide on whether to establish a link based on the existing network. Examples for deterministic network formation are Kranton and Minehart (2001) and Elliott (2011a).

[13] See also Kircher (2009) and Galeanos and Kircher (2009) and Fontaine (2004).

will of course depend on the wage (or more generally on the wage mechanism) it posts. If we make the labor market large in the usual way, by letting $u, v \to \infty$ with $v/u = \theta$, the number of applications are distributed according to a Poisson distribution with mean a/θ.

Different wage mechanisms will generate different distributions of networks and different matchings. In equilibria where firms post mechanisms that imply the same expected payoff, it is optimal for risk neutral workers to randomize between firms. If firms, however, post mechanisms that imply different expected payoffs, equilibrium requires that low-wage firms (who make more profit per worker, if they hire a worker) receive less applications (so that they are less likely to hire) than high-wage firms. Albrecht, Gautier and Vroman (2006) for example show in a directed search framework where firms post an auction with a minimum bid that all firms post the same wage (equal to the reservation wage), while Galeanos and Kircher (2009) and Kircher (2009) show that, if firms post fixed wages and commit not to Bertrand compete ex post, firms post different wages. Below, we show that wage dispersion is socially not efficient in terms of network formation.

4.3.1 Social planner's problem

An unconstrained social planner will trivially assign each unemployed workers to a vacancy such that the number of matches equals the short side of the market. If workers send out multiple applications, the same first best assignment can be achieved, if the social planner partitions the labor market into submarkets where the number of firms and workers in each submarket is no higher than the number of applications. However, if the social planner faces the same coordination frictions as the market, he must assign symmetric strategies to identical workers implying that he can only decide about the probability with which a worker sends an application to a subgroup of firms.

We constrain the social planner to choose the set of firm-subgroups C (where each subgroup c is defined by a certain color), the measure of vacancies v_c within each subgroup c and the probability $p_{c,i}$ that a worker sends its i-th application to subgroup $c \in C$. The expected number of applications sent to subgroup c is equal to

$$a_c = u \sum\nolimits_{i=1}^{a} p_{c,i}.$$

The total number of workers u_c that applies to subgroup c can be less than the total number of links (or applications a_c) between firms of subgroup c and unemployed workers, if workers send more than one application to one subgroup. The total number of workers that applied to subgroup c is given by $u_c = \left(1 - \prod_{i=1}^{a} (1 - p_{c,i})\right) u$, where $\prod_{i=1}^{a} (1 - p_{c,i})$

equals the probability that a given unemployed worker does not send any application to subgroup c. While vacancies can by definition only be part of one subgroup, workers can be linked to at most a different subgroups depending on where they send their applications to. Workers are, however, only part of one subgraph (worker-, firm- or even subgraph). Subgraphs can, therefore, contain vacancies of different subgroups, if the workers that belong to that subgraph are linked to vacancies in different subgroups.

The maximum matching that is achieved by ex-post competition implies that the number of matches within each subgroup c equals the number of workers in firm subgraphs u_c^f, the number of firms in worker subgraphs v_c^w and the number of firms (or workers) in even subgraphs v_c^e or (u_c^e) i.e. $M_c = u_c^f + v_c^w + v_c^e$. Using the fact that the sum of vacancies equals the sum of vacancies in firm-, worker- and even subgraphs, i.e. $v_c = v_c^f + v_c^w + v_c^e$, we can rewrite the expected number of matches in a subgroup c as the number of vacancies in subgroup c minus the number of vacancies in subgroup c in firm subgraphs that are not matched, i.e.,

$$M_c = v_c - \left(v_c^f - u_c^f\right). \tag{1}$$

Coromina-Bosch's Decomposition Theorem allows us also to derive the first derivatives of the matching function with respect to an additional application.[14] Since all firms in worker- and even subgraph are matched, only applications to firms in firm subgraphs can result in additional matches. In addition, an application will only lead to an *additional* match, if the worker who sends the application is not part of an even or firm subgraph (since all workers in even or firm subgraphs are already matched). In other words the worker must be part of a worker subgraph. The probability that a vacancy is part of a firm subgraph in subgroup c is v_c^f/v_c and the probability that a worker is part of a worker subgraphs is u^w/u, where u^w is the number of unemployed workers in worker subgraphs. An additional application of a randomly selected worker therefore leads with the following probability to an additional match,

$$\frac{\triangle M_c}{\triangle a_c} = \frac{v_c^f}{v_c}\frac{u^w}{u}. \tag{2}$$

Any additional match that is formed by a link of a vacancy in a firm subgraph and a worker in a worker subgraph decreases the excess number of firms in firm subgraphs, i.e. decreases $v_c^f - u_c^f$. If the excess number of firms in a particular subgrapth is equal to one, then this additional match turns vacancies located in firm subgraphs into vacancies in

[14] Note, that a marginal increase in the expected number of applications results form a marginal increase in the application probability $p_{c,i}$.

even-subgraphs. Thus, an additional link decreases the expected number of firms in firm subgraphs. Furthermore, any additional match that is formed by a link between a vacancy in a firm subgraph and a worker in a worker subgraph reduces the number of workers in worker subgraphs u^w. This implies that the number of matches in any subgroup c is a concave function of the number of applications, i.e.,

$$\frac{\triangle^2 M_c}{\triangle a_c^2} = \frac{1}{v_c}\frac{u^w}{u}\frac{\triangle v_c^f}{\triangle a_c} + \frac{v_c^f}{v_c}\frac{1}{u}\frac{\triangle u^w}{\triangle a_c} < 0, \text{ since } \frac{\triangle v_c^f}{\triangle a_c} < 0 \text{ and } \frac{\triangle u^w}{\triangle a_c} < 0. \tag{3}$$

Although we do not know the exact form of the matching function, these properties of the matching function are sufficient to characterize the necessary and sufficient conditions for efficient network formation.

The social planner chooses the set of firm-subgroups C, the measure of firms v_c within each subgroup c and the total number of applications a_c that unemployed workers send to vacancies in each subgroup c, i.e.,

$$\max_{C, v_c, a_c} \sum\nolimits_{c \in C} M_c.$$

Note, that choosing the total number of applications a_c is (by the law of large numbers) equivalent to choosing the probability $p_{c,i}$ that a worker sends its i-th application to subgroup c, since symmetry requires that all workers use the same application strategy.

Theorem 2: *(i) Network formation is efficient, if and only if*

$$\frac{v_c^f}{v_c} = \frac{v^f}{v} \quad \textit{for all} \quad c \in C. \tag{4}$$

which is equivalent to having the same application intensity in each subgroup, i.e.,

$$\frac{v_c}{a_c} = \frac{v}{au} \quad \textit{for all} \quad c \in C. \tag{5}$$

(ii) Efficient network formation is independent of the set C of subgroups and the number of vacancies v_c in each subgroup.

Proof: We prove part (i) by showing that the number of matches is only maximized, if $v_c^f/v_c = v^f/v$ for all $c \in C$. Suppose that the probability of a firm being in a firm subgraph is higher in the red subgroup $r \in C$ than in the blue subgroup $b \in C$, i.e. $v_r^f/v_r > v_b^f/v_b$.[15] This implies according to equation (2) the following relationship for the

[15] Note, if no worker applied to subgroup c, then all firms in subgroup c are in firm-subgraphs, i.e. $v_c^f/v_c = 1$.

marginal matches generated by an additional application, i.e.,

$$\frac{v_r^f}{v_r} > \frac{v_b^f}{v_b} \Longleftrightarrow \frac{\triangle M_r}{\triangle a_r} > \frac{\triangle M_b}{\triangle a_b}.$$

Given that the matching function is concave in the number of applications, see equation (3), the total number of matches in subgroups r and b can be increased by redirecting applications from subgroup b to subgroup r, until

$$\frac{\triangle M_r}{\triangle a_r} = \frac{\triangle M_b}{\triangle a_b} \Longleftrightarrow \frac{v_r^f}{v_r} = \frac{v_b^f}{v_b}.$$

Since the same argument applies for all $c \in C$, condition (4) must hold in order to maximize the total number of matches for a given set of subgroups C.

Condition (4) holds, because the number of applications a_c directed to each subgroup is adjusted accordingly. This implies that the number of applications to each subgroup is proportional to the number of vacancies in each subgroup, i.e.,

$$\frac{a_c}{v_c} = \frac{au}{v} \quad \text{for all} \quad c \in C.$$

To prove part (ii) we show that conditional on $v_c^f/v_c = v^f/v$ and $a_c/v_c = au/v$ for all $c \in C$, the total number of matches is independent of the number of subgroups C and the number of vacancies v_c within each subgroup. If market tightness is the same in all subgroups, i.e. condition (5) holds by symmetry, the number of unemployed workers matched with vacancies in each subgroup must also be proportional to the number of vacancies in each subgroup. This is also true for each subtype of matched workers, i.e., for workers in worker-, firm- and even subgraphs. Thus, the number of matched workers u_c^f that are part of firm subgraphs in subgroup c are proportional to the number of vacancies in subgroup c, i.e.,

$$\frac{u_c^f}{v_c} = \frac{u^f}{v} \quad \text{for all} \quad c \in C.$$

Using this last equality and condition (4) implies that the total number of matches is independent of the set C of subgroups and the number of vacancies v_c in each subgroup, i.e.,

$$\begin{aligned}
\sum\nolimits_{c\in C} M_c &= \sum\nolimits_{c\in C} \left[v_c - \left(v_c^f - u_c^f\right)\right] \\
&= \sum\nolimits_{c\in C} v_c \left[1 - \left(\frac{v_c^f}{v_c} - \frac{u_c^f}{v_c}\right)\right] \\
&= \left[1 - \left(\frac{v^f}{v} - \frac{u^f}{v}\right)\right] \sum\nolimits_{c\in C} v_c \\
&= v - \left(v^f - u^f\right).
\end{aligned}$$

where the third step applies equality (4).■

The efficiency condition for network formation in Theorem 2 implies that all vacancies should have the same probability to be contacted by a worker. This makes the network as balanced as possible and therefore minimizes the fraction of firms that are not matched. Shimer (2005) derives a similar condition for a directed search environment where workers can apply to only one firm. In the setting by Galeanos and Kircher (2009), where workers can send more than one application but firms can contact only one worker, the total number of matches is also maximized, if all firms have the same probability to be contacted by a worker. In contrast, the efficiency condition in Kircher (2009), where workers send multiple applications and firms can contact all workers, differs from our efficiency condition, because he constraints the social planner to let a subgroup of firms always match first (i.e. be in a high location). Those firms in a high location should be more likely to be contacted by a worker, since this reduces the probability that a worker is not available for hiring at a firm in a low location (where firms can only match, if their candidates do not have an offer from a firm in a high location). Allowing the social planner to also choose the network clearing mechanism, Corollary 2 shows that it is not optimal to let a subgroup of firms match first. Thus, Kircher's (2009) efficiency result differs from our efficiency result, because he restricts the social planner to use a network clearing mechanism that does not allow for ex post Bertrand competition.

The second part of Theorem 2 also implies that the total number of matches does not change, if there are no firm subgroups. The simulated examples in the next section show that this property only holds for a large number of workers and firms. If the labor market is small, the expected number of matches decreases, if firms are partitioned into different subgroups. Thus, random search, where no subgroups exist, generates a socially efficient distribution of networks for any market size.

Corollary 3: *Random search leads to efficient network formation.*

Random search leads to evenly distributed links between workers and firms and therefore minimizes the expected number of workers in worker subgraphs and the expected number of firms in firm subgraphs. A large part of firms in firm subgraphs are firms without an application.[16] The following Proposition shows when this event is least likely.

[16] As we will show in the next section, if a is small relatively to u and v, and $\theta = 1$, this is the main reason for a firm not to hire a worker.

Proposition 1: *If workers fully randomize, the fraction of vacancies without applicants is minimized.*

Proof. See Appendix B.

Although random search leads to efficient network formation, Theorem 2 does not directly imply that directed search with different posted wages and ex-post competition leads to inefficient network formation. Using Lemma 4, however, implies that the equal profit condition for high- and low-wage firms violates condition (4), which is necessary to get efficient network formation.

Proposition 2: *If equally productive firms post different wages, network formation is not efficient.*

Proof: We prove this Proposition by showing that the equal profit condition, which must hold if equally productive firms post different wages, implies $v_L^f/v_L > v_H^f/v_H$, if $w_H > w_L$. Lemma 4 implies that all firms earn zero profit, if they are part of a firm subgraph, since they pay a wage equal to the worker's marginal product. High wage firms in even or worker subgraphs earn $1 - w_H$. Low wage firms earn more. If they are part of an even (or worker-)subgraph, they pay with probability π^e (or π^w) their low posted wage w_L and with probability $1-\pi^e$ (or $(1-\pi^w)$) the high posted wage w_H. Note that the appropriate probabilities satisfy $\pi > 0$, since there exists a positive probability that a low wage firm does not have to compete with a high wage firm for a worker in an even or a worker subgraph.

The **equal profit condition** of high and low wage firms is, therefore, given by

$$\begin{aligned} & \frac{v_H^f}{v_H}[1-1] + \left[\frac{v_H^e}{v_H} + \frac{v_H^w}{v_H}\right][1-w_H] \\ = \; & \frac{v_L^f}{v_L}[1-1] + \frac{v_L^e}{v_L}[1-\pi^e w_L - (1-\pi^e)\,w_H] + \frac{v_L^w}{v_L}[1-\pi^w w_L - (1-\pi^w)\,w_H] \end{aligned}$$

Rearranging and noting that $\frac{v_c^f}{v_c} + \frac{v_c^e}{v_c} + \frac{v_c^w}{v_c} = 1$ implies

$$\left[\frac{v_L^f}{v_L} - \frac{v_H^f}{v_H}\right][1-w_H] = \left[\frac{v_L^w}{v_L}\pi^w + \frac{v_L^e}{v_L}\pi^e\right][w_H - w_L]\,.$$

Since $w_H > w_L$, it follows immediately that $v_L^f/v_L > v_H^f/v_H$. ■

4.3.2 Simulations

To illustrate that randomization is desirable when agents do not know the network we numerically compare randomization with the case where a subset of the vacancies has a higher arrival rate of applications. The details of our algorithm are given in Appendix C. The basic steps of the algorithm are as follows. First, we color a fraction q of the vacancies blue and a fraction $(1-q)$ green and let each worker send one application to a blue vacancy and the other $(a-1)$ applications to a green one. Each blue vacancy receives an application from worker 1 with probability $1/qv$ and the same for workers 2,..., u. For the $a=3$ example, each green vacancy gets with probability, $(a-1)/(1-q)v$, the second application of worker 1 and if it did not get the second one, it gets the third one with probability $(a-2)/((1-q)v-1)$ etc. The same holds for the other workers. For $q = 1/a$, the arrival rate at each firm is the same and the only difference to full randomization is that the market is partitioned. Since we want to focus on network formation here, we assume maximum matching on each realized network. If for example blue vacancies would have a priority in matching (e.g. if they offer higher wages) as in Kircher (2009), the number of matches could be lower than we report here.

a	p_n	$E(M)$	var(M)	I/v	J/u
joint					
2	1.343	10.416	0.812	0.012	0.061
3	0.377	11.554	0.382	0.045	0.343
6	0.003	11.997	0.003	0.000	0.003
partitioned ($q=\frac{1}{3}$)					
2	1.748	10.064	0.875	0.187	0.684
3	**0.405**	**11.533**	**0.387**	**0.046**	**0.347**
6	0.124	11.876	0.111	0.010	0.122
partitioned ($q=\frac{1}{6}$)					
2	2.851	9.137	0.945	0.242	0.675
3	0.719	11.206	0.540	0.075	0.510
6	**0.005**	**11.995**	**0.005**	**0.000**	**0.005**

Table 2: Simulation results for $v=u=12$

Let the fraction of firms in firm subgraphs be I/v and the fraction of workers in worker subgraphs be J/u. Let p_n be the probability that a firm receives no workers, and finally let var(M) be the variance of applicants that a particular firm receives. Below we present simulation results for $v = u = 12$. We generate a sample of 1000 networks for each case. In Table 2 below, we present the probability that a firm receives no workers p_n, the mean and variance of the number of matches M, the average number of firms in firm subgraphs and the average number of workers in worker subgraphs for different values of a, q.

We see that partitioning the market reduces the expected number of matches but that for $q = \frac{1}{a}$ (those rows are in bold), the arrival rate at each firm is the same and the difference with the fully random case is relatively small. We also see that if a is large relatively to v, that partitioning hardly matters. Firms are swamped with applications and almost all firms and workers are connected, implying that the number of matches is close to 12.

Figure 5 below, shows the distribution of matches for the case where all firms are part of one group (i.e., workers fully randomize) and Figure 6 shows the case where 1/3 of the vacancies are blue and each worker sends one of their applications to a blue vacancy.

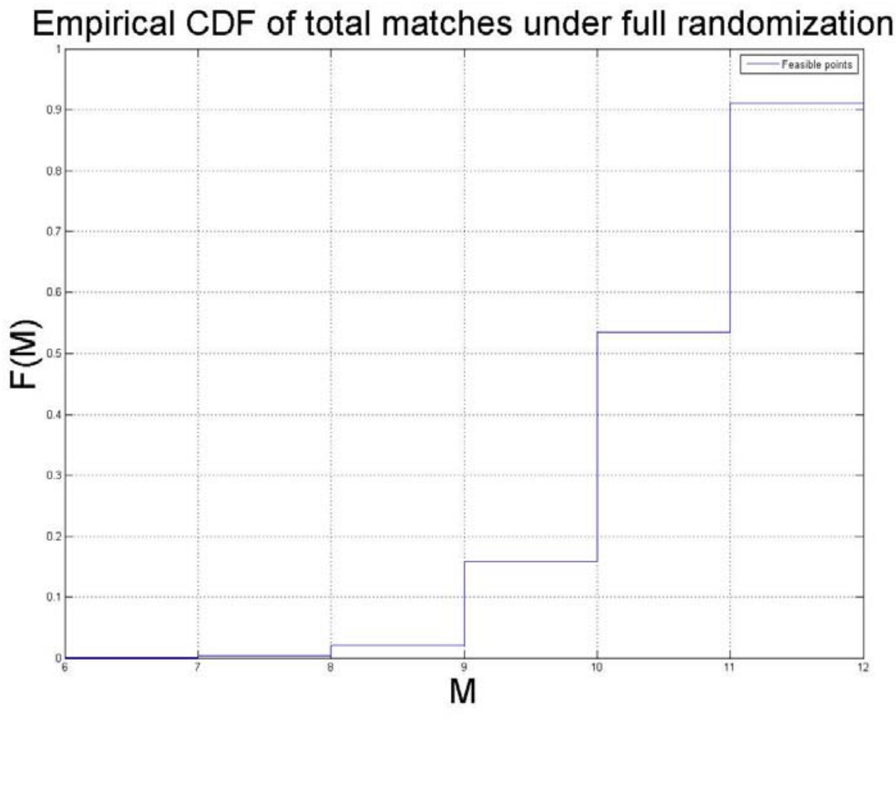

Figure 5: $u = v = 12, a = 2$

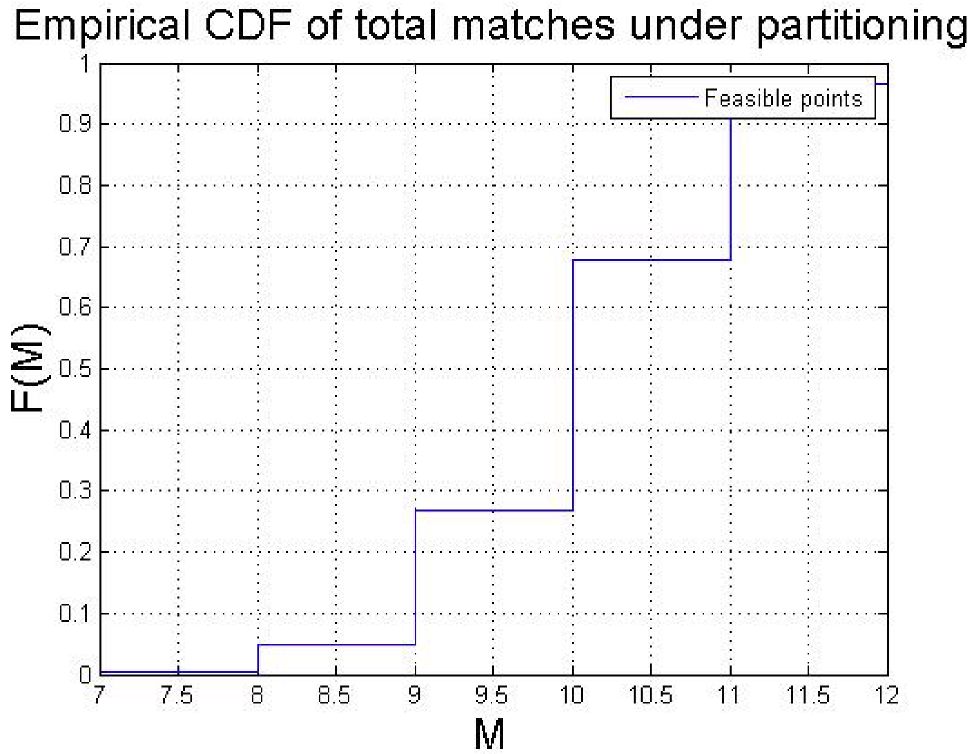

Figure 6: $u = v = 12, a = 2, q = 1/3$

We find that the cdf in the full randomization case first order stochastically dominates the one in the partitioning case. Under randomization, the probability that less than 11 matches are formed is about 70% while under randomization this is only about 50%.

5 Final remarks

This paper contributes to one of the fundamental question in economics namely under which conditions do decentralized markets generate constraint efficient outcomes. Our focus is on the labor market where it is common that unemployed workers simultaneously send multiple applications which creates a bipartite network between workers and firms. In such an environment there are two coordination frictions, (i) workers do not know where other workers apply to and (ii) firms do not know which workers are considered by other firms. We show that the second coordination friction between firms can be eliminated, if wages in the decentralized market are determined by ex-post Bertrand competition and if firms can go back and forth between their applicants. The number of matches on a given network that is formed, if workers send multiple applications to firms, equals the maximum matching possible. The first coordination friction is minimized if the decentralized market ensures that workers apply to each vacancy with equal probability. This implies that an equilibrium with wage dispersion is inefficient in terms of network formation.

Although a wage mechanism that has ex-post Bertrand competition and no wage dispersion is efficient in terms of network formation and clearing, it will most likely not be efficient in other dimensions like vacancy creation and search intensity (number of applications). Kircher (2009) shows for example that wage commitment without ex post competition implies wage dispersion and that the resulting equilibrium is efficient in terms of search intensity and firm entry. Combining those results suggests that there may not exist a wage mechanism that by itself generates the constrained efficient outcome.

An important and interesting extension for future research is to allow for heterogeneity in firm and or worker types, see Shimer (2005). We conjecture that this makes ex post Bertrand competition equally desirable as in a homogenous firm world, because high productive firms should be able to outbid low productive firms. Furthermore, this will make directed search more desirable than in our setting because high productive firms should be able to signal their types in order to get matched with a higher probability.

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6 Appendix

A Derivations 3 by 3 example

In this appendix we characterize the full recall equilibrium and determine the number of matches for a simple economy with 3 unemployed workers and 3 firms, each with one vacancy ($u = v = 3$). As we stated in section 3, firms do not know the realization of the network but only observe how many candidates they have and they can go to their candidates back and forth with higher wage offers as often as they like (complete recall). Workers have a reservation wage of 0 and a matched firm-worker pair produces 1.

If workers apply to 1 or all 3 jobs, all wage mechanisms that we consider generate the same distribution of networks and they clear equally efficient. Therefore, we focus on the case where workers send two applications ($a = 2$). We take the number of applications and the number of firms as given but it is important to note that if they were to be endogenized, an equilibrium with $a = 2$ and $v = 3$ only exists for a particular set of application and entry cost. Only for the random-search with Bertrand competition case, this equilibrium will not exist without a positive minimum wage because the payoff for a worker would be 0 in this 3 by 3 example. This can simply be solved by introducing a minimum wage $w = \underline{w}$. [17] We consider the following cases. First, we look at random search: (i) with ex-post Bertrand competition, and (ii) with wage posting and no Bertrand competition. Then, we look at directed search models where workers observe the posted wages ex-ante and they can direct their applications to a particular wage. Also in this case, we consider: (i) ex-post Bertrand competition, and (ii) wage commitment and no Bertrand competition. The maximum number of matches according to Hall's marriage

[17] Gautier and Moraga-Gonzalez (2005) and Albrecht et al. (2006) find that workers send too many applications because of rent seeking and that entry is excessive because firms have too much market power. Kircher (2009) shows that with directed search and full recall, entry and search intensity are socially efficient.

market theorem is $23/8 \simeq 2.889$. This is realized under random search with Bertrand competition. In this case there is no wage dispersion and we get a maximum matching on each realized network. Under directed search with Bertrand competition, the expected number of matches is slightly less (2.884) because there is a little bit of wage dispersion (we conjecture that this disappears in a larger market). Random search with wage posting generates 2.703 matches on average. The lack of Bertrand competition leads to fewer than the maximum number of matches on all network realizations. Directed search with commitment generates even more wage dispersion than the directed search with ex-post Bertrand competition. The ex-ante wage dispersion and the lack of Bertrand competition leads to fewer than the maximum number of matches on a given network. On average 2.538 matches are realized in that case.

A.1 Random search with ex-post competition

This case has been studied in Gautier and Wolthoff (2009) without recall and with heterogeneous firms. In a random search equilibrium, where firms are able to compete ex-post, firms will first offer a wage equal to the workers' reservation wage or to the legal minimum wage, $w = \underline{w}$. Since all firms are identical and Bertrand competition assures that on each realized network the maximum number of matches is realized, a firm only fails to match if it has no candidates. So the matching probability is $\left(1 - (1 - 2/3)^3\right)$ and the aggregate number of matches is given by

$$M = 3\left(1 - \left(1 - \frac{2}{3}\right)^3\right) = \frac{26}{9} = 2.889.$$

Note that in this 3 by 3 example this equilibrium is hard to sustain with positive application cost and without a legal minimum wage. Absent a minimum wage, the worker's payoff is zero because firms who have 2 or 3 candidates will not have to increase their initial bids. With a sufficiently high minimum wage, even workers with a positive application cost will participate.

A.2 Random search equilibrium with wage commitment

This case has been studied before by Gautier and Moraga Gonzalez (2005) with limited and full recall. Since they also give a 3 by 3 example we just summarize their results here. For the same reasons as in Burdett Judd (1983) and Burdett Mortensen (1998) there exists no symmetric pure-strategy equilibrium in wages. For any candidate equilibrium wage or mass point below the marginal product, there exists a profitable ε upward deviation that gives a discrete jump in payoffs (because the deviant always wins the race against rivals who stick to that candidate equilibrium). The wage distribution is determined by a firm-indifference condition. The expected payoff for a firm offering w while the other firms offer a wage from $F(w)$ is given by (see Gautier and Moraga Gonzalez (2005) for a derivation),

$$\pi_i(w; F(w)) = \left[p^1 F^2(w) + 2p^2 F(w)\left[1 - F(w)\right] + p^3 \left[1 - F(w)\right]^2\right](1 - w) \tag{6}$$

where p^1, p^2 and p^3 are the probabilities that a firm offering the highest, second highest and lowest wage in the market hires a worker, which are derived in Gautier and Moraga Gonzalez (2004). The lowest wage that is offered must be the worker's reservation wage or the minimum wage implying that $F(\underline{w}) = 0$ so $\pi_i(\underline{w}; F(\underline{w})) = p^3 = \pi_i(w; F(w))$ where the last equality follows from the equal profit condition. This can be used to solve for $F(w)$ in (6). If workers send out 2 applications then firms choose wages from the set $[\underline{w}, \frac{21\underline{w}+5}{26}]$ according to the cumulative wage distribution.

$$F(w) = \frac{4}{3} - \frac{1}{3}\sqrt{16 - 63\frac{(w - \underline{w})}{(1 - w)}} \tag{7}$$

In equilibrium firms receive an expected payoff of $\pi_i = \frac{21}{27}$ and workers get a job with probability 73/81 and each firm receives an expected payoff of $\pi_i = \frac{21}{27}(1 - \underline{w})$. The total number of matches is $M = \frac{73}{27} = 2.703$. The lack of ex post competition makes the expected number of matches less than the maximum.

A.3 Directed search with ex-post competition

A similar wage setting scheme has been studied before by Albrecht, Gautier and Vroman (2006) who considered no recall and limited short lists. Short lists, however, do not allow firms to go back to their candidates. Thus, firms that compete ex-post, but shortlist their candidates will generally not achieve the maximum matching. Below, we derive the full recall equilibrium.

First, we consider a candidate equilibrium where 2 firms (L) offer a low and one (H) offers a high wage. This equilibrium does exist. In both cases we must calculate the expected payoffs of the firms (both H and L) and the workers. Below, we derive the distribution of possible networks, and the payoffs for the different firms and the workers for each network realization for the LLH case. In the second part, we show that an equilibrium where one firms offers a low wage and two firms a high wage does not exist.

A.3.1 Two low and one high wage firm

Let H be the high wage firm and A and B the low wage firms, ξ is the probability that a worker sends one of her applications to H. Finally, denote the payoffs of firm type $j \in \{H, L\}$ by π^{Fj}.

The first three columns of Table 3 below denote the number of applicants at each firm (H,A or B). The fourth column gives the probability that the allocation occurs. The remaining columns contain the profit of firms (H, A and B).

If the high wage firm has an applicant its payoff is given by $(1-w^h)$, otherwise it is zero. A low wage firm only has to increase its posted wage, if its candidate also received an offer from the high wage firm. This can only occur if firm (H) has 3 candidates (else the other low wage firm has 3 candidates implying that the worker in question cannot also receive an offer from firm (H)). A low wage firm, e.g. firm (A), receives 0 if it has no candidates, it receives $(1-w)$ if $(H, A, B) \neq (3, 1, 2)$ and it receives $(2/3)(1-w) + (1/3)(1-w^h)$ if $(H, A, B) = (3, 1, 2)$. This last expression follows from the fact that with probability 1/3, its candidate receives an offer from the high wage firm so firm (A) has to increase its initial offer in order to be able to employ the worker. Table 3 below summarizes the expected payoffs of the firms.

H	A	B	probability	π^{FH}	$\pi^{FL}(A)$	$\pi^{FL}(B)$
3	1	2	$\frac{3}{8}\xi^3$	$1-w^h$	$\frac{2}{3}(1-w)+\frac{1}{3}\left(1-w^h\right)$	$1-w$
3	2	1	$\frac{3}{8}\xi^3$	$1-w^h$	$1-w$	$\frac{2}{3}(1-w)+\frac{1}{3}\left(1-w^h\right)$
3	3	0	$\frac{1}{8}\xi^3$	$1-w^h$	$1-w$	0
3	0	3	$\frac{1}{8}\xi^3$	$1-w^h$	0	$1-w$
2	2	2	$\frac{2}{4}3\xi^2(1-\xi)$	$1-w^h$	$1-w$	$1-w$
2	3	1	$\frac{1}{4}3\xi^2(1-\xi)$	$1-w^h$	$1-w$	$1-w$
2	1	3	$\frac{1}{4}3\xi^2(1-\xi)$	$1-w^h$	$1-w$	$1-w$
1	3	2	$\frac{1}{2}3\xi(1-\xi)^2$	$1-w^h$	$1-w$	$1-w$
1	2	3	$\frac{1}{2}3\xi(1-\xi)^2$	$1-w^h$	$1-w$	$1-w$
0	3	3	$(1-\xi)^3$	0	$1-w$	$1-w$

Table 3: Network formation and firm payoffs with two low and one high-wage firm

Next, we turn to the worker payoffs. First, consider the initial allocation after the first two workers have applied (H2,A2,B2) $= (2, 1, 1)$. These are shown in the first three columns. The next column shows the probability that this allocation occurs. The next three columns (H,A,B) denote the final allocation and the eighth column shows the conditional probability that this allocation occurs. The last column shows the payoff of a worker, who send one application to the high wage firm and one to the low wage firm. This is the notation we use throughout the paper. Call the worker in question, worker 1. In case of $(3,2,1)$, which is equivalent to $(3,1,2)$ by replacing A by B, worker 1 is with two other applicants at firm H and with one other applicant at a low-wage firm. With probability 1/3, the worker's application at the high-wage firm is selected and she receives w^h. With probability 2/3 she is not selected first at the high-wage firm. In that case, worker 2 or 3 is alone and the high-wage firm could have selected the worker at firm B with probability 1/2 (this firm will bid $w^h+\varepsilon$ and gets the worker, the high-wage firm can afford not to bid further) and in that case the worker in question (1) is picked next by the high-wage firm with probability 1/2 and receives w^h. In all other cases worker 1 is matched at the low-wage firm for sure and receives w.

$$\frac{1}{3}w^h + \frac{2}{3}\left(\frac{1}{2}\frac{1}{2}w^h + \frac{3}{4}w\right) = \frac{1}{2}w + \frac{1}{2}w^h$$

In case of an initial allocation $(2,2,0)$ or $(2,0,2)$, we have to distinguish between the outcomes $(3,3,0)$ and $(3,0,3)$ on the one hand and $(3,2,1)$ and $(3,1,2)$ on the other hand. Clearly, in case $(3,3,0)$ or $(3,0,3)$ worker one is picked with probability 1/3 by the high-wage firm. If not, which occurs with probability 2/3, the worker is one of two remaining workers at firm H, i.e., she is hired with probability 1/2 at wage w. In the $(3,2,1)$ and $(3,1,2)$ cases, the worker is offered the wage w^h with probability 1/3 by the high-wage firm. With probability 2/3 she is not selected first at the high-wage firm. In that case, worker 2 and 3 are at the other low-wage firm. Since this firm can be sure that it will always hire, it will not increase its bid and the first worker will be hired by the high-wage firm. Consequently, there is no chance for worker 1 to get his wage bid up if she is not first at the high-wage firm.

In case of an initial allocation $(1,1,2)$ or $(1,2,1)$ we have to distinguish between the final outcomes $(2,2,2)$ on the one hand and $(2,3,1)$ or $(2,1,3)$ on the other hand. Consider the $(2,2,2)$ case. With probability 1/2, the worker receives an offer from the high-wage firm and receives w^h. What happens if he does not receive the first offer from the high-wage firm? With probability 1/2, the other worker, who is at the high-wage firm, receives an offer at a low-wage firm. The low-wage firm will, however, not bid more, because it knows it will always get a worker so this other worker will be hired by the high-wage firm and we have,

$$\frac{1}{2}w^h + \frac{1}{2}w.$$

Next consider the $(2,1,3)$ case which is equivalent to the $(2,3,1)$ case. Again, with probability $1/2$, the worker receives an offer from the high-wage firm and receives w^h. The only low-wage firm that would be willing to compete is the firm with one worker. But this worker cannot have applied to the high-wage firm because there is a low wage firm with 3 candidates. So the payoffs are,

$$\frac{1}{2}w^h + \frac{1}{2}w.$$

Finally consider the $(1,3,2)$ case which is equivalent to the $(1,2,3)$ case. If there are no other workers at the high-wage firm, the worker in question gets w^h for sure. All of the above is summarized in the following table.

H2	A2	B2	probability	H	A	B	probability	π^{WH}
2	1	1	$\frac{1}{2}\xi^2$	3	2	1	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{1}{2}w$
				3	1	2	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{1}{2}w$
2	2	0	$\frac{1}{4}\xi^2$	3	3	0	$\frac{1}{2}$	$\frac{1}{3}w + \frac{1}{3}w^h$
				3	2	1	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{2}{3}w$
2	0	2	$\frac{1}{4}\xi^2$	3	1	2	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{2}{3}w$
				3	0	3	$\frac{1}{2}$	$\frac{1}{3}w + \frac{1}{3}w^h$
1	1	2	$\xi\,(1-\xi)$	2	2	2	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{1}{2}w$
				2	1	3	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{1}{2}w$
1	2	1	$\xi\,(1-\xi)$	2	3	1	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{1}{2}w$
				2	2	2	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{1}{2}w$
0	2	2	$(1-\xi)^2$	1	3	2	$\frac{1}{2}$	w^h
				1	2	3	$\frac{1}{2}$	w^h

Table 4: Worker payoffs with two low and one high-wage firm for a worker who one application to a low-wage firm and one to a high-wage firm

Next, we must consider the payoff for a worker who sends both applications to the low wage firms. This worker cannot receive more than w because she did not apply to a high-wage firm and because the event that both low-wage firms have 1 candidate has a zero probability. So we have,

This just implies that a worker who sends both applications to a low-wage firm receives expected utility of $\left(1-(1-\xi)^2\right)w + (1-\xi)^2\,\frac{2}{3}w$.

Equilibrium In equilibrium all firms must make equal profits regardless of the wage they post, i.e. $\pi^{FH} = \pi^{FL}$. Workers send their application such that they maximize their

H2	A2	B2	probability	H	A	B	π^{WL}
2	1	1	$\frac{1}{2}\xi^2$	2	2	2	w
2	2	0	$\frac{1}{4}\xi^2$	2	3	1	w
2	0	2	$\frac{1}{4}\xi^2$	2	1	3	w
1	1	2	$\xi(1-\xi)$	1	2	3	w
1	2	1	$\xi(1-\xi)$	1	3	2	w
0	2	2	$(1-\xi)^2$	0	3	3	$\frac{2}{3}w$

Table 5: Worker payoffs with two low and one high-wage firm for a worker who sends both applications to low-wage firms

expected payoff. Given that at most two different wages, i.e. $w^h \geq w$, are offered by three firms. Workers are either indifferent between sending one of their applications to the high-wage firm and the other one to one of the low-wage firms (mixed strategy equilibrium), i.e. $\pi^{WH} = \pi^{WL}$, or they strictly prefer to send one application to the high-wage firm and send the second one to one of the low wage firms (pure strategy equilibrium), i.e. $\pi^{WH} > \pi^{WL}$. Note, that sending both applications to both low-wage firms cannot be an equilibrium, since it would violate the equal profit condition. Given the workers' optimal application strategy firms must have no incentive to deviate from the wages offered in equilibrium.

Using tables 4 and 5, the worker's indifference condition can be written as

$$\begin{aligned}
& \frac{1}{2}\xi^2\left(\frac{1}{2}\left(\frac{1}{2}w^h+\frac{1}{2}w\right)+\frac{1}{2}\left(\frac{1}{2}w^h+\frac{1}{2}w\right)\right)+\frac{1}{4}\xi^2\left(\frac{1}{2}\left(\frac{1}{3}w^h+\frac{1}{3}w\right)+\frac{1}{2}\left(\frac{1}{3}w^h+\frac{2}{3}w\right)\right) \\
& +\frac{1}{4}\xi^2\left(\frac{1}{2}\left(\frac{1}{3}w^h+\frac{2}{3}w\right)+\frac{1}{2}\left(\frac{1}{3}w^h+\frac{1}{3}w\right)\right)+\xi(1-\xi)\frac{1}{2}2\left(\frac{1}{2}w^h+\frac{1}{2}w\right) \\
& +\xi(1-\xi)\left(\frac{1}{2}\left(\frac{1}{2}w^h+\frac{1}{2}w\right)+\frac{1}{2}\left(\frac{1}{2}w^h+\frac{1}{2}w\right)\right)+(1-\xi)^2\left(\frac{1}{2}\left(w^h\right)+\frac{1}{2}\left(w^h\right)\right) \\
= & \left(1-(1-\xi)^2\right)w+(1-\xi)^2\frac{2}{3}w,
\end{aligned}$$

which simplifies to

$$\begin{aligned}
\frac{5}{12}w^h\xi^2+w\xi-\frac{1}{2}w\xi^2-w^h\xi+w^h &= w\left(-\frac{1}{3}\xi^2+\frac{2}{3}\xi+\frac{2}{3}\right) \\
\left(\frac{5}{12}\xi^2-\xi+1\right)w^h &= \left(\frac{1}{6}\xi^2-\frac{1}{3}\xi+\frac{2}{3}\right)w.
\end{aligned}$$

$$\frac{w^h}{w}=\frac{\frac{1}{6}\xi^2-\frac{1}{3}\xi+\frac{2}{3}}{\frac{5}{12}\xi^2-\xi+1}. \tag{8}$$

Since $w^h \geq w$, an equilibrium can only exist for $\xi \geq 2/3$. Totally differentiating equation (8) with respect to w^h and w gives

$$\begin{aligned} \frac{d\xi}{dw^h} &= \frac{\frac{5}{12}\xi^2 - \xi + 1}{w\left(\frac{2}{6}\xi - \frac{1}{3}\right) - w^h\left(\frac{10}{12}\xi - 1\right)} && (9) \\ \frac{d\xi}{dw} &= \frac{\frac{1}{6}\xi^2 - \frac{1}{3}\xi + \frac{2}{3}}{w^h\left(\frac{10}{12}\xi - 1\right) - w\left(\frac{2}{6}\xi - \frac{1}{3}\right)}. && (10) \end{aligned}$$

The equal profit condition implies $\pi^{FH} = \pi^{FA}$, i.e.

$$\begin{aligned} \left(1 - w^h\right)\left(1 - (1-\xi)^3\right) &= \frac{1}{8}\xi^3\left(1 - w^h\right) + \left(1 - \frac{1}{4}\xi^3\right)(1 - w) \\ \left(1 - w^h\right) &= \frac{\left(1 - \frac{1}{4}\xi^3\right)(1 - w)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)} \\ \frac{\left(1 - w^h\right)}{(1 - w)} &= \frac{\left(1 - \frac{1}{4}\xi^3\right)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)}. && (11) \end{aligned}$$

Again, since $w^h \geq w$, an equilibrium can only exist for $\xi \geq 2/3$.

A low wage firm has no incentive to offer a different wage, if

$$\frac{d\pi^{FL}}{dw} = -\left(1 - \frac{1}{4}\xi^3\right) + \frac{3}{8}\xi^2\left(1 - w^h\right)\frac{d\xi}{dw} - \frac{3}{4}\xi^2(1 - w)\frac{d\xi}{dw} = 0.$$

Use (10) to eliminate $\frac{d\xi}{dw}$ and eliminate w^h by using (11) yields,

$$\frac{\left(\frac{3}{8}\xi^2\left(1 - w^h\right) - \frac{3}{4}\xi^2(1 - w)\right)\left(\frac{1}{6}\xi^2 - \frac{1}{3}\xi + \frac{2}{3}\right)}{w^h\left(\frac{10}{12}\xi - 1\right) - w\left(\frac{2}{6}\xi - \frac{1}{3}\right)} = \left(1 - \frac{1}{4}\xi^3\right)$$

$$\begin{aligned} &\frac{\left(\frac{1}{6}\xi^2 - \frac{1}{3}\xi + \frac{2}{3}\right)}{\left(1 - \frac{1}{4}\xi^3\right)}\left(\frac{\frac{3}{8}\xi^2\left(1 - \frac{1}{4}\xi^3\right)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)} - \frac{3}{4}\xi^2\right)(1 - w) \\ &= \left(\frac{10}{12}\xi - 1\right) - \frac{\left(1 - \frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi - 1\right)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)}(1 - w) - w\left(\frac{2}{6}\xi - \frac{1}{3}\right) \end{aligned}$$

$$w = \frac{\frac{\left(\frac{1}{6}\xi^2 - \frac{1}{3}\xi + \frac{2}{3}\right)}{\left(1 - \frac{1}{4}\xi^3\right)}\left(\frac{\frac{3}{8}\xi^2\left(1 - \frac{1}{4}\xi^3\right)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)} - \frac{3}{4}\xi^2\right) + \frac{\left(1 - \frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi - 1\right)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)} - \left(\frac{10}{12}\xi - 1\right)}{\frac{\left(\frac{1}{6}\xi^2 - \frac{1}{3}\xi + \frac{2}{3}\right)}{\left(1 - \frac{1}{4}\xi^3\right)}\left(\frac{\frac{3}{8}\xi^2\left(1 - \frac{1}{4}\xi^3\right)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)} - \frac{3}{4}\xi^2\right) + \frac{\left(1 - \frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi - 1\right)}{\left(1 - \frac{1}{8}\xi^3 - (1-\xi)^3\right)} - \left(\frac{2}{6}\xi - \frac{1}{3}\right)}. \quad (12)$$

The high-wage firm firm has no incentive to offer a different wage, if

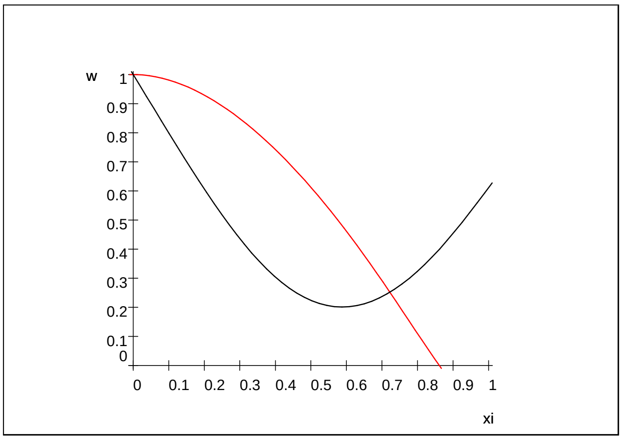
$$\frac{d\pi^{FH}}{dw^h} = -\left(1 - (1-\xi)^3\right) + 3(1-\xi)^2\left(1 - w^h\right)\frac{d\xi}{dw^h} = 0$$

Replace $\dfrac{d\xi}{dw^h}$ by the rhs of (9) and eliminate w^h by using (11) yields,

$$\left(1-(1-\xi)^3\right) = 3\,(1-\xi)^2\,\frac{\frac{\left(1-\frac{1}{4}\xi^3\right)(1-w)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}\left(\frac{5}{12}\xi^2-\xi+1\right)}{w\left(\frac{2}{6}\xi-\frac{1}{3}\right)-\left(1-\frac{\left(1-\frac{1}{4}\xi^3\right)(1-w)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}\right)\left(\frac{10}{12}\xi-1\right)}$$

$$w = \frac{\frac{3(1-\xi)^2\left(1-\frac{1}{4}\xi^3\right)\left(\frac{5}{12}\xi^2-\xi+1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)\left(1-(1-\xi)^3\right)}-\frac{\left(1-\frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi-1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}+\left(\frac{10}{12}\xi-1\right)}{\frac{3(1-\xi)^2\left(1-\frac{1}{4}\xi^3\right)\left(\frac{5}{12}\xi^2-\xi+1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)\left(1-(1-\xi)^3\right)}-\frac{\left(1-\frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi-1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}+\left(\frac{2}{6}\xi-\frac{1}{3}\right)}. \tag{13}$$

The plot below, shows this equation (optimality condition for high wage firms) together with the optimality condition for low wage firms (12). We see that there is a unique feasible strictly positive (ξ, w) pair that satisfies both equilibrium conditions.



The equilibrium (ξ, w) pair can be found by substituting the wage using equations (12) and (13). This implies the following equation for the application probability to the high-wage firm,

$$\frac{\frac{\left(\frac{1}{6}\xi^2-\frac{1}{3}\xi+\frac{2}{3}\right)}{\left(1-\frac{1}{4}\xi^3\right)}\left(\frac{\frac{3}{8}\xi^2\left(1-\frac{1}{4}\xi^3\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}-\frac{3}{4}\xi^2\right)+\frac{\left(1-\frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi-1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}-\left(\frac{10}{12}\xi-1\right)}{\frac{\left(\frac{1}{6}\xi^2-\frac{1}{3}\xi+\frac{2}{3}\right)}{\left(1-\frac{1}{4}\xi^3\right)}\left(\frac{\frac{3}{8}\xi^2\left(1-\frac{1}{4}\xi^3\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}-\frac{3}{4}\xi^2\right)+\frac{\left(1-\frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi-1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}-\left(\frac{2}{6}\xi-\frac{1}{3}\right)}$$

$$= \frac{\frac{3(1-\xi)^2\left(1-\frac{1}{4}\xi^3\right)\left(\frac{5}{12}\xi^2-\xi+1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)\left(1-(1-\xi)^3\right)}-\frac{\left(1-\frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi-1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}+\left(\frac{10}{12}\xi-1\right)}{\frac{3(1-\xi)^2\left(1-\frac{1}{4}\xi^3\right)\left(\frac{5}{12}\xi^2-\xi+1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)\left(1-(1-\xi)^3\right)}-\frac{\left(1-\frac{1}{4}\xi^3\right)\left(\frac{10}{12}\xi-1\right)}{\left(1-\frac{1}{8}\xi^3-(1-\xi)^3\right)}+\left(\frac{2}{6}\xi-\frac{1}{3}\right)}$$

$$\begin{aligned} & \left(736\xi - 1008\xi^2 + 432\xi^3 + 80\xi^4 - 48\xi^5 - 54\xi^6 + 21\xi^7 - 192\right) \\ & \left(84\xi^2 - 288\xi + 300\xi^3 - 430\xi^4 + 267\xi^5 - 80\xi^6 + 9\xi^7 + 144\right) \\ = & \left(352\xi - 336\xi^2 + 32\xi^3 + 260\xi^4 - 216\xi^5 + 46\xi^6 - 192\right) \\ & \left(1092\xi^3 - 204\xi^2 - 288\xi - 1330\xi^4 + 798\xi^5 - 243\xi^6 + 30\xi^7 + 144\right) \end{aligned}$$

Solving yields $\xi = 0.722$, which implies that $w = 0.252$, and $w^h = 0.272$. The expected number of hirings is obtained by summing up the expected hiring probability of the high-wage firm and both low-wage firms, i.e.,

$$M = \left(1 - (1-\xi)^3\right) + 2\left(\frac{1}{8}\xi^3 + \left(1 - \frac{1}{4}\xi^3\right)\right) = 2.884$$

A.3.2 No equilibrium exists where two firms offer a high wage or all firms offer the same wage

Firms' payoffs We show that no equilibrium exists where 2 firms post w and one firm $w^l \leq w$. In that case, the payoffs for the firms can be calculated as follows.[18] First note that in this case the high-wage firms receive zero, if they have no candidate, but if they have at least one candidate they always receive $1-w$, because they never have to compete up to one since it cannot be that two firms have only one applicant.

Next, consider the payoffs for a low-wage firm. If it has 3 candidates, it will get $1-w^l$. If $(L, A, B) = (2, 2, 2)$, the low-wage firm has to pay $w \geq w^l$ if both its workers receive an offer from both high-wage firms in the first round. The probability that this happens is $1/2 \times 1/2$. Thus, the payoff is

$$\frac{3}{4}(1-w^l) + \frac{1}{4}(1-w).$$

Next consider the $(1, 3, 2)$ case, which is equivalent to the $(1, 2, 3)$ case. Suppose worker 1 applied to the low-wage firm. The low-wage firm has to Bertrand compete, if worker 1 is first at the high-wage firm with 3 applications (probability 1/3) or else (probability 2/3), if both high-wage firms offer the job to the same worker (probability 1/2) in round 1 (the high-wage firm with only 2 applications will get the worker) and if worker 1 gets an offer from the high-wage firm with initially 3 applications in the second round (probability 1/2). Note that the low-wage firm will bid most aggressively, i.e. will

[18] An alternative proof that only rules out an equilibrium without wage dispersion is available upon request.

offer $w+\varepsilon$, and will get the worker for sure. So its payoff is

$$\left(\frac{1}{3}+\frac{2}{3}\left(\frac{1}{2}\frac{1}{2}\right)\right)(1-w)+\frac{1}{2}\left(1-w^{l}\right)=\frac{1}{2}\left(1-w^{l}\right)+\frac{1}{2}(1-w).$$

The Table below summarizes the expected payoffs of the firms.

L	A	B	probability	π^{FL}	π^{FH}, A	π^{FH}, B
3	1	2	$\frac{3}{8}\xi^{3}$	$1-w^{l}$	$1-w$	$1-w$
3	2	1	$\frac{3}{8}\xi^{3}$	$1-w^{l}$	$1-w$	$1-w$
3	3	0	$\frac{1}{8}\xi^{3}$	$1-w^{l}$	$1-w$	0
3	0	3	$\frac{1}{8}\xi^{3}$	$1-w^{l}$	0	$1-w$
2	2	2	$\frac{2}{4}3\xi^{2}(1-\xi)$	$\frac{3}{4}(1-w^{l})+\frac{1}{4}(1-w)$	$1-w$	$1-w$
2	3	1	$\frac{1}{4}3\xi^{2}(1-\xi)$	$1-w^{l}$	$1-w$	$1-w$
2	1	3	$\frac{1}{4}3\xi^{2}(1-\xi)$	$1-w^{l}$	$1-w$	$1-w$
1	3	2	$\frac{1}{2}3\xi(1-\xi)^{2}$	$\frac{1}{2}(1-w^{l})+\frac{1}{2}(1-w)$	$1-w$	$1-w$
1	2	3	$\frac{1}{2}3\xi(1-\xi)^{2}$	$\frac{1}{2}(1-w^{l})+\frac{1}{2}(1-w)$	$1-w$	$1-w$
0	3	3	$(1-\xi)^{3}$	0	$1-w$	$1-w$

Table 6: Network formation and firm payoffs with one low and two high-wage firms

Workers' payoffs First, consider the payoff for the marginal worker when she sends one of her applications to the low-wage firm. Consider the initial allocation after the first two workers have applied (L2,A2,B2) $=(2,1,1)$. Let us call the worker in question, worker 1. In case of $(3,2,1)$, which is equivalent to $(3,1,2)$ by replacing A by B, worker 1 is with two other applicants at firm L and with one other applicant at a high-wage firm. The worker prefers now an offer from a high-wage firm. With probability $1/2$ she gets the job offer at the high-wage in the first round. With probability $1/2$ she is not selected first at the high-wage firm. In this case the other worker, who also sends her second application to the low-wage firm, prefers an offer by the high-wage firm. Thus, the other worker will accept the first offer by the high-wage firm. Subsequently, worker 1 will only receive w^{l}. Therefore, her expected payoffs are, $\frac{1}{2}w+\frac{1}{2}w^{l}$.

In case of an initial allocation $(2,2,0)$ or $(2,0,2)$, we must distinguish between the outcomes $(3,3,0)$ and $(3,0,3)$ on the one side and $(3,2,1)$ and $(3,1,2)$ on the other side.

Clearly, in case $(3,3,0)$ or $(3,0,3)$ worker one is picked with probability $1/3$ by the high-wage firm. If not, which occurs with probability $2/3$, then the worker is one of two remaining workers at a the low-wage firm, i.e. she his hired with probability $1/2$ at the wage w^l. In case of $(3,2,1)$ and $(3,1,2)$ the worker is the only applicant at the high-wage firm. She therefore receives the wage w for sure.

In case of an initial allocation $(1,1,2)$ or $(1,2,1)$ we have to distinguish between the final outcomes $(2,2,2)$ on the one hand and $(2,3,1)$ or $(2,1,3)$ on the other hand. Consider the $(2,2,2)$ case. Worker 1 gets the higher wage w, if the high-wage firm offers her the job. With probability $1/2$ worker 1 is chosen first by the high-wage firm. If worker 1 is not chosen in the first round (which happens with probability $1/2$), then with probability $1/2$ both high-wage firms will compete for the same worker. The high-wage firm, where worker 1 applied, remains vacant with probability $1/2$. In this case the high-wage firm offer worker 1 the wage w in the second round. Thus, worker 1 gets the payoff

$$\frac{1}{2}w + \frac{1}{2}\left(\frac{1}{2}\frac{1}{2}w + \frac{3}{4}w^l\right) = \frac{5}{8}w + \frac{3}{8}w^l.$$

Next consider the $(2,1,3)$ case. If worker 1 applied to the low-wage firm she is at firm B with probability 1 and selected with probability $1/3$. If she is not selected (probability $2/3$), the worker who is alone at firm A is selected by firm B (probability $1/2$) and hired by firm A with probability 1 (firm A will bid most aggressively). In that case worker 1 will be hired by firm B with probability $1/2$. Else she will be hired by the low-wage firm. So she receives in expectation,

$$\frac{1}{3}w + \frac{2}{3}\left(\frac{1}{2}\frac{1}{2}w + \frac{3}{4}w^l\right) = \frac{1}{2}w + \frac{1}{2}w^l.$$

Finally, consider the $(1,3,2)$ cases. Worker 1, who applied to firm L must have also applied to firm A, because A got 3 applications. Worker 1 gets hired at the high-wage firm A with probability $1/3$, or (probability $(2/3)$ if firm A and B offer the job to the same worker (probability $1/2$) then firm B hires this worker (firm B will bid more aggressively), and worker 1 then gets an offer at the firm with 3 applicants (A) with probability $1/2$ (the low-wage firm will bid more aggressive and hire the worker so he ends up there at wage w). Else he gets w^l. So his expected payoffs are,

$$\frac{1}{3}w + \frac{2}{3}\left(\frac{1}{2}\frac{1}{2}w + \frac{3}{4}w^l\right) = \frac{1}{2}w + \frac{1}{2}w^l$$

The following table summarizes all of the above.

L2	A2	B2	probability	L	A	B	probability	π^{WL}
2	1	1	$\frac{1}{2}\xi^2$	3	2	1	$\frac{1}{2}$	$\frac{1}{2}w+\frac{1}{2}w^l$
				3	1	2	$\frac{1}{2}$	$\frac{1}{2}w+\frac{1}{2}w^l$
2	2	0	$\frac{1}{4}\xi^2$	3	3	0	$\frac{1}{2}$	$\frac{1}{3}w+\frac{1}{3}w^l$
				3	2	1	$\frac{1}{2}$	w
2	0	2	$\frac{1}{4}\xi^2$	3	1	2	$\frac{1}{2}$	w
				3	0	3	$\frac{1}{2}$	$\frac{1}{3}w+\frac{1}{3}w^l$
1	1	2	$\xi(1-\xi)$	2	2	2	$\frac{1}{2}$	$\frac{5}{8}w+\frac{3}{8}w^l$
				2	1	3	$\frac{1}{2}$	$\frac{1}{2}w+\frac{1}{2}w^l$
1	2	1	$\xi(1-\xi)$	2	3	1	$\frac{1}{2}$	$\frac{1}{2}w+\frac{1}{2}w^l$
				2	2	2	$\frac{1}{2}$	$\frac{5}{8}w+\frac{3}{8}w^l$
0	2	2	$(1-\xi)^2$	1	3	2	$\frac{1}{2}$	$\frac{1}{2}w+\frac{1}{2}w^l$
				1	2	3	$\frac{1}{2}$	$\frac{1}{2}w+\frac{1}{2}w^l$

Table 7: Worker payoffs with one low and two high-wage firms

Finally, a worker who sends both applications to high-wage firms will be hired with probability $2/3$ in the $(0,3,3)$ case. Else this worker will be hired for sure and receives w. The Table below gives the payoff to a worker who sends both applications to the high-wage firms.

L2	A2	B2	probability	L	A	B	π^{WH}
2	1	1	$\frac{1}{2}\xi^2$	2	2	2	w
2	2	0	$\frac{1}{4}\xi^2$	2	3	1	w
2	0	2	$\frac{1}{4}\xi^2$	2	1	3	w
1	1	2	$\xi(1-\xi)$	1	2	3	w
1	2	1	$\xi(1-\xi)$	1	3	2	w
0	2	2	$(1-\xi)^2$	0	3	3	$\frac{2}{3}w$

Table 8: Worker payoffs with one low and two high-wage firms for a worker who sends both applications to high-wage firms

Equilibrium In equilibrium all firms must make equal profits regardless of the wage they post, i.e. $\pi^{FL}=\pi^{FA}=\pi^{FB}$. Workers send their application such that they maximize their expected payoff, given that at most two different wages, i.e., $w^l < w$, are offered by three firms. Workers are indifferent between sending one of their applications to the low-wage firm and the other one to one of the high-wage firms (mixed strategy equilibrium), if $\pi^{WL} = \pi^{WH}$.

Using the payoff tables, the indifference condition can be written as,

$$\begin{aligned}
& \frac{1}{2}\xi^2\left(\frac{1}{2}\left(\frac{1}{2}w+\frac{1}{2}w^l\right)+\frac{1}{2}\left(\frac{1}{2}w+\frac{1}{2}w^l\right)\right)+\frac{1}{4}\xi^2\left(\frac{1}{2}\left(\frac{1}{3}w+\frac{1}{3}w^l\right)+\frac{1}{2}w\right) \\
& +\frac{1}{4}\xi^2\left(\frac{1}{2}w+\frac{1}{2}\left(\frac{1}{3}w+\frac{1}{3}w^l\right)\right)+\xi\left(1-\xi\right)\left(\frac{1}{2}\left(\frac{5}{8}w+\frac{3}{8}w^l\right)+\frac{1}{2}\left(\frac{1}{2}w+\frac{1}{2}w^l\right)\right) \\
& +\xi\left(1-\xi\right)\left(\frac{1}{2}\left(\frac{1}{2}w+\frac{1}{2}w^l\right)+\frac{1}{2}\left(\frac{5}{8}w+\frac{3}{8}w^l\right)\right) \\
& +\left(1-\xi\right)^2\left(\frac{1}{2}\left(\frac{1}{2}w+\frac{1}{2}w^l\right)+\frac{1}{2}\left(\frac{1}{2}w+\frac{1}{2}w^l\right)\right) \\
= & \left(1-\left(1-\xi\right)^2\right)w+\left(1-\xi\right)^2\frac{2}{3}w.
\end{aligned}$$

Simplifying yields,

$$\begin{aligned}
\frac{1}{2}w-\frac{1}{24}w^l\xi^2+\frac{1}{8}w\xi-\frac{1}{24}w\xi^2-\frac{1}{8}w^l\xi+\frac{1}{2}w^l &= \left(1-\left(1-\xi\right)^2\right)w+\left(1-\xi\right)^2\frac{2}{3}w \\
\left(-\frac{1}{24}\xi^2-\frac{1}{8}\xi+\frac{1}{2}\right)w^l &= \left(-\frac{7}{24}\xi^2+\frac{13}{24}\xi+\frac{1}{6}\right)w.
\end{aligned}$$

$$\frac{w^l}{w} = \frac{-\frac{7}{24}\xi^2+\frac{13}{24}\xi+\frac{1}{6}}{-\frac{1}{24}\xi^2-\frac{1}{8}\xi+\frac{1}{2}} \tag{14}$$

Since $w^l < w$, an equilibrium can only exist for $\xi < 2/3$. Totally differentiating equation (14) with respect to w^l and w gives

$$\frac{d\xi}{dw^l} = \frac{-\frac{1}{24}\xi^2-\frac{1}{8}\xi+\frac{1}{2}}{w\left(-\frac{14}{24}\xi+\frac{13}{24}\right)-w^l\left(-\frac{2}{24}\xi-\frac{1}{8}\right)} \tag{15}$$

$$\frac{d\xi}{dw} = \frac{-\frac{7}{24}\xi^2+\frac{13}{24}\xi+\frac{1}{6}}{w^l\left(-\frac{2}{24}\xi-\frac{1}{8}\right)-w\left(-\frac{14}{24}\xi+\frac{13}{24}\right)}. \tag{16}$$

Given the workers' optimal application strategy firms must have no incentive to deviate from the wages offered in equilibrium. Using the payoff tables, the firm payoffs can be written as,

$$\begin{aligned}\pi^{FL} &= \left(\xi^3+\frac{3}{2}\xi^2(1-\xi)+\frac{9}{8}\xi^2(1-\xi)+\frac{3}{2}\xi(1-\xi)^2\right)(1-w^l)\\ &\quad+\left(\frac{3}{8}\xi^2(1-\xi)+\frac{3}{2}\xi(1-\xi)^2\right)(1-w)\\ &= \left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)(1-w^l)+\left(\frac{3}{2}\xi-\frac{21}{8}\xi^2+\frac{9}{8}\xi^3\right)(1-w)\end{aligned}$$

$$\begin{aligned}\pi^{FH} &= \left(\frac{7}{8}\xi^3+3\xi^2(1-\xi)+3\xi(1-\xi)^2+(1-\xi)^3\right)(1-w)\\ &= \left(1-\frac{1}{8}\xi^3\right)(1-w)\end{aligned}$$

The equal profit condition implies $\pi^{FL}=\pi^{FH}$ i.e.

$$\begin{aligned}\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)(1-w^l) &= \left(1-\frac{3}{2}\xi+\frac{21}{8}\xi^2-\frac{5}{4}\xi^3\right)(1-w)\\ (1-w^l) &= \frac{\left(1-\frac{3}{2}\xi+\frac{21}{8}\xi^2-\frac{5}{4}\xi^3\right)}{\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)}(1-w)\end{aligned}$$

$$\frac{(1-w^l)}{(1-w)}=\frac{\left(\frac{7}{8}\xi^3+\frac{21}{8}\xi^2(1-\xi)+\frac{3}{2}\xi(1-\xi)^2+(1-\xi)^3\right)}{\left(\xi^3+\frac{3}{2}\xi^2(1-\xi)+\frac{9}{8}\xi^2(1-\xi)+\frac{3}{2}\xi(1-\xi)^2\right)}$$

High-wage firms have no incentive to offer a different wage, if

$$\frac{d\pi^{FA}}{dw}=-\left(1-\frac{1}{8}\xi^3\right)-\frac{3}{8}\xi^2\frac{d\xi}{dw}=0$$

Use (16) to eliminate $\frac{d\xi}{dw}$ gives,

$$-\frac{\frac{3}{8}\xi^2\left(-\frac{7}{24}\xi^2+\frac{13}{24}\xi+\frac{1}{6}\right)}{w^l\left(-\frac{2}{24}\xi-\frac{1}{8}\right)-w\left(-\frac{14}{24}\xi+\frac{13}{24}\right)}=\left(1-\frac{1}{8}\xi^3\right)$$

Use (14) to eliminate w^l, yields,

$$w=\frac{\frac{\left(1-\frac{3}{2}\xi+\frac{21}{8}\xi^2-\frac{5}{4}\xi^3\right)\left(-\frac{2}{24}\xi-\frac{1}{8}\right)}{\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)}-\frac{\frac{3}{8}\xi^2\left(-\frac{7}{24}\xi^2+\frac{13}{24}\xi+\frac{1}{6}\right)}{\left(1-\frac{1}{8}\xi^3\right)}-\left(-\frac{2}{24}\xi-\frac{1}{8}\right)}{\frac{\left(1-\frac{3}{2}\xi+\frac{21}{8}\xi^2-\frac{5}{4}\xi^3\right)\left(-\frac{2}{24}\xi-\frac{1}{8}\right)}{\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)}-\left(-\frac{14}{24}\xi+\frac{13}{24}\right)} \tag{17}$$

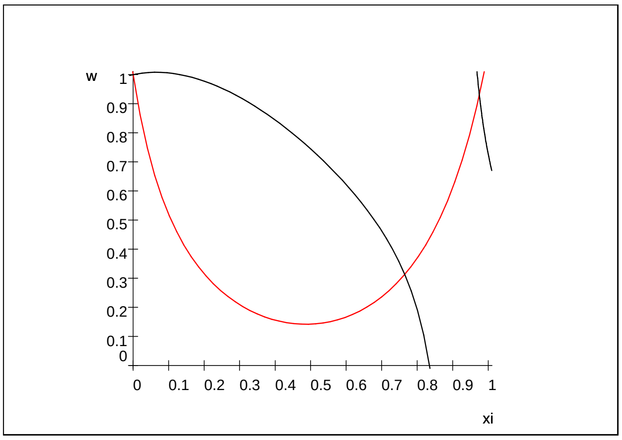
The low-wage firm firm has no incentive to offer a different wage, if

$$\begin{aligned}
\frac{d\pi^{FL}}{dw^l} &= -\left(\frac{3}{2}\xi - \frac{3}{8}\xi^2 - \frac{1}{8}\xi^3\right) \\
&+ \left(\left(\frac{3}{2} - \frac{3}{4}\xi - \frac{3}{8}\xi^2\right)(1-w^l) + \left(\frac{3}{2} - \frac{21}{4}\xi + \frac{27}{8}\xi^2\right)(1-w)\right)\frac{d\xi}{dw^l} \\
&= 0
\end{aligned}$$

Use(15) to eliminate the $\frac{d\xi}{dw^l}$ term and (14) to eliminate w^l, yields,

$$w = \frac{\frac{\left(\frac{3}{2}-\frac{3}{4}\xi-\frac{3}{8}\xi^2\right)\left(1-\frac{3}{2}\xi+\frac{21}{8}\xi^2-\frac{5}{4}\xi^3\right)}{\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)} + \left(\frac{3}{2} - \frac{21}{4}\xi + \frac{27}{8}\xi^2\right) + \frac{\left(\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)-\left(1-\frac{3}{2}\xi+\frac{21}{8}\xi^2-\frac{5}{4}\xi^3\right)\right)\left(-\frac{2}{24}\xi-\frac{1}{8}\right)}{\left(-\frac{1}{24}\xi^2-\frac{1}{8}\xi+\frac{1}{2}\right)}}{\frac{\left(\frac{3}{2}-\frac{3}{4}\xi-\frac{3}{8}\xi^2\right)\left(1-\frac{3}{2}\xi+\frac{21}{8}\xi^2-\frac{5}{4}\xi^3\right)}{\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)} + \left(\frac{3}{2} - \frac{21}{4}\xi + \frac{27}{8}\xi^2\right) + \frac{\left(-\frac{14}{24}\xi+\frac{13}{24}\right)\left(\frac{3}{2}\xi-\frac{3}{8}\xi^2-\frac{1}{8}\xi^3\right)}{\left(-\frac{1}{24}\xi^2-\frac{1}{8}\xi+\frac{1}{2}\right)}}$$

The Figure below plots this equation (black) together with equation (17) (red) and shows that there exists no equilibrium wage that satisfies $\xi < 2/3$. Thus, an equilibrium with one low and two high wage firms cannot exist.



A.4 Wage posting with commitment

This case has been studied before by Kircher (2009) and for the no-recall case by Galeanos and Kircher (2010). Assume that firms ex ante commit to paying a wage and that they do not (or cannot) engage in ex post Bertrand competition, even, if it is profitable to do so after observing the number of applicants. Workers have a desire to diversify their application portfolios and they choose them according to a marginal improvement algorithm, see Chade and Smith (2006). The first application is sent to a location that generates the highest expected payoff, the second one to the location that gives the greatest marginal improvement given the first one, etcetera. Firms respond to the worker's desire to diversify by offering different wages. Suppose two firms (A and B) post a wage w and the third firm posts w^j. Since firms commit to their wage, they always earn one minus the wage posted, if they are matched. Workers observe the posted wages and send their applications in order to maximize their utility. Let ξ be the probability that a worker sends one application to the single firm that offers a different wage. If the wage of this firm is lower, we call this firm L and if it is higher, we call it H. The parameter ξ depends on w^j and w through an indifference condition, which we develop below. We focus again on the case where workers send two applications.

A.4.1 Existence of an equilibrium with two low and one high-wage firm

First, we determine the profit of the high and the low wage firms. In equilibrium all firms must make (i) equal profits and (ii) have no incentive to deviate. In the next step, we determine the workers' payoffs. At the equilibrium application probability ξ, workers must be indifferent between applying to the high and to the low wage firms. In Galeanos and Kircher (2009) and Kircher (2009) the application probability to one set of firms is always one, implying that the workers' indifference condition need not hold with equality. In addition Galeanos and Kircher (2009) and Kircher (2009) allow for firm entry. This ensures that the profits of both types of firms are driven down to zero, i.e., the equal profit condition holds with equality and allows firms to post the wages that maximize workers' utility. The 3 by 3 example considered here does not allow for firm entry. This implies that workers will use a mixed application strategy, i.e., $\xi \neq 1$.

Firm payoffs The firm that offers the high wage, i.e., $w^h > w$, is always matched conditional on receiving at least one application.

If all firms receive 2 applications, i.e., in case of $(2, 2, 2)$, unlike the case with Bertrand competition, a low-wage firm will not hire with certainty because it can lose both its candidates to one of the other firms. The worker who is selected first by the low-wage firm also applied to the high wage firm with probability $1/2$ and the worker will go to the high-wage firm if she is first there (which occurs with probability $1/2$). The low-wage firm in question will not hire in that case, if its second applicant that must have applied to the other low-wage firm is first at the other low-wage firm (probability $1/2$) and accepts that offer (probability $1/2$). Alternatively, the same worker could be first at both low-wage firms (with probability $1/2 \times 1/2$). If the worker accepts at the other low-wage firm (probability $1/2$), the low-wage firm remains unmatched, if the second worker is first at the high-wage firm (probability $1/2$).

$$1 - \left(\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\right) = \frac{7}{8}.$$

If a low-wage firm has one applicant then it is always competing with the firm with 3 applicants. If the high-wage firm has 3 applicants, as in the $(3,1,2)$ case, then the low-wage firm remains unmatched, if the worker is first at the high-wage firm (probability $1/3$), since $w^h > w$. Thus, the matching probability of a low-wage firm firm is $2/3$. If the other low-wage firm has 3 applications, then the low-wage firm remains unmatched, if the worker is first at the other low-wage firm (probability $1/3$) and accepts the offer (probability $1/2$). Thus, the hiring probability of the low-wage firm is $5/6$.

Table 9 summarizes the expected matching probabilities for each realized network for the high-wage firm and both low-wage firms and the likelihood of occurrence.

H	A	B	probability	π^{FH}	$\pi^{FL}(A)$	$\pi^{FL}(B)$
3	1	2	$\frac{3}{8}\xi^3$	1	$\frac{2}{3}$	1
3	2	1	$\frac{3}{8}\xi^3$	1	1	$\frac{2}{3}$
3	3	0	$\frac{1}{8}\xi^3$	1	1	0
3	0	3	$\frac{1}{8}\xi^3$	1	0	1
2	2	2	$\frac{2}{4}3\xi^2(1-\xi)$	1	$\frac{7}{8}$	$\frac{7}{8}$
2	3	1	$\frac{1}{4}3\xi^2(1-\xi)$	1	1	$\frac{5}{6}$
2	1	3	$\frac{1}{4}3\xi^2(1-\xi)$	1	$\frac{5}{6}$	1
1	3	2	$\frac{1}{2}3\xi(1-\xi)^2$	1	1	1
1	2	3	$\frac{1}{2}3\xi(1-\xi)^2$	1	1	1
0	3	3	$(1-\xi)^3$	0	1	1

Table 9: Firm matching probabilities with one highe- and two low-wage firms

The respective profits of the high- and the low-wage firms are given by

$$\begin{aligned}\pi^{FH} &= \left(1-w^h\right)\left(\xi^3 + 3\xi^2(1-\xi) + 3\xi(1-\xi)^2\right) \\ &= \left(1-w^h\right)\left(3\xi - 3\xi^2 + \xi^3\right) \qquad (18)\end{aligned}$$

$$\begin{aligned}
\pi^{FL} &= (1-w)\left(\frac{3}{8}\xi^3\frac{2}{3}+\frac{1}{2}\xi^3+\frac{2}{4}3\xi^2(1-\xi)\frac{7}{8}+\frac{1}{4}3\xi^2(1-\xi)\left(1+\frac{5}{6}\right)\right) \\
&\quad +(1-w)\left(3\xi(1-\xi)^2+(1-\xi)^3\right) \\
&= (1-w)\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right). \qquad (19)
\end{aligned}$$

The equal profit condition implies,

$$\frac{\left(1-w^h\right)}{(1-w)}=\frac{\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1}{3\xi-3\xi^2+\xi^3} \qquad (20)$$

The equal profit condition also requires $\frac{1-w^h}{1-w}<1$ or

$$\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1<3\xi-3\xi^2+\xi^3$$

This is satisfied for $\xi>0.558$. Thus, at this stage we cannot rule out a pure- (application) strategy equilibrium for workers, i.e. $\xi=1$. In this case the indifference condition for workers need not hold with equality. The only requirement is that sending an application to the high wage firm and one to a low-wage firm gives at least the same payoff as sending both applications to the low wage firms. There could, however, also be a mixed strategy equilibrium. In this case workers need to be indifferent between the two strategies. In any equilibrium, it must be the case that firms have no incentive to deviate.

Worker payoffs Table 10 shows in columns 1 to 3 the possible networks that can arise. Suppose that worker 1 sends one application to the high-wage firm and one application to either firm A or B. The resulting network is then given by columns 5 to 7.

In the $(2,1,1)$ network, worker 1 is with two other applicants at firm H and with one other applicant at a low-wage firm. Since the other applicant also applied to firm H, worker 1 is always matched. With probability 1/3, the high-wage firm offers the worker the job (at $w^h>w$), which he accepts. Otherwise worker 1 is hired at the low-wage firm at w.

In the $(2,2,0)$ or $(2,0,2)$ case, we must distinguish between the outcomes $(3,3,0)$ and $(3,0,3)$ on the one hand and $(3,2,1)$ and $(3,1,2)$ on the other hand. Clearly, in the $(3,3,0)$ or $(3,0,3)$ cases, worker 1 is picked with probability 1/3 by the high-wage firm. If not, which occurs with probability 2/3, then the worker is one of two remaining workers at the low-wage firm, i.e. she is hired with probability 1/2. In case of $(3,2,1)$ and $(3,1,2)$ the worker is offered a wage w by the low-wage firm with certainty. Thus, the worker accept the wage offer w^h at the high-wage firm if she is the first one chosen (probability 1/3), otherwise (probability 2/3) she accepts wage w at the low-wage firm.

In the $(1,1,2)$ or $(1,2,1)$ cases we must distinguish between the final outcomes $(2,2,2)$ on the one hand and $(2,3,1)$ or $(2,1,3)$ on the other hand. If the final network is given by $(2,2,2)$, worker 1 either gets an offer from the high-wage firm with probability 1/2 and she will accept this offer. If worker 1 is second at the high-wage firm, there is no chance that worker 1 still gets an offer from that high-wage firm, since the first worker will accept for sure. However, there is a chance that worker 1 gets an offer from the low-wage firm. Worker 1 gets an offer from the low-wage firm, if she is first (probability 1/2), or if the first worker at the low-wage firm does not accept the offer from that low-wage firm. Note first that it is impossible that the first worker at the low-wage firm applied to the high-wage firm, since this would imply that both worker 1 and the first worker would have applied to the same firms. However, the outcome $(2,2,2)$ would then imply that the third worker sent both applications to the same firm. This cannot be true. Thus, the first worker at the low-wage does not accept the offer, if he receives an offer from the other low-wage firm (which happens with probability 1/2 and she accepts with probability 1/2). Her expected payoff in that case is therefore,

$$\frac{1}{2}w^h + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2}\right)w = \frac{1}{2}w^h + \frac{5}{16}w$$

If the final network is given by $(2,3,1)$ or $(2,1,3)$ the worker receives with probability 1/2 an offer from the high-wage firm firm and accepts it. Note, that if worker 1 is second at the high-wage firm, worker 1 has only one competing worker at the low-wage firm with 3 candidates. Worker 1 gets the job at the low-wage firm, if she is chosen first (probability 1/2) or if she is second and the first worker accepts an offer at the other low-wage firm (probability 1/2)

$$\frac{1}{2}w^h + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\frac{1}{2}\right)w = \frac{1}{2}w^h + \frac{3}{8}w$$

Finally, in the $(0,2,2)$ case, worker 1 will be the only candidate at the high-wage firm H and receive the wage w^h with certainty.

Table 11 presents the worker's payoffs, if she sends both applications to the low wage firms. If the final network is given by $(2,2,2)$, worker 1 gets at least one offer from a low-wage firm, because one of the workers that applied to the high-wage firm will accept the high-wage firm's offer (since $w^h > w$) and thus, worker 1 will be the only remaining applicant at the low-wage firm where the worker that got a job at the high-wage firm sent his second application to. In case of $(2,3,1)$ or $(2,1,3)$, worker 1 is the only worker

H2	A2	B2	probability	H	A	B	probability	π^{WH}
2	1	1	$\frac{1}{2}\xi^2$	3	2	1	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{2}{3}w$
				3	1	2	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{2}{3}w$
2	2	0	$\frac{1}{4}\xi^2$	3	3	0	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{1}{3}w$
				3	2	1	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{2}{3}w$
2	0	2	$\frac{1}{4}\xi^2$	3	1	2	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{2}{3}w$
				3	0	3	$\frac{1}{2}$	$\frac{1}{3}w^h + \frac{1}{3}w$
1	1	2	$\xi(1-\xi)$	2	2	2	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{5}{16}w$
				2	1	3	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{3}{8}w$
1	2	1	$\xi(1-\xi)$	2	3	1	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{3}{8}w$
				2	2	2	$\frac{1}{2}$	$\frac{1}{2}w^h + \frac{5}{16}w$
0	2	2	$(1-\xi)^2$	1	3	2	$\frac{1}{2}$	w^h
				1	2	3	$\frac{1}{2}$	w^h

Table 10: Worker payoffs in the directed search with commitment case for a worker who sends one application to a high and one to a low-wage firm

at firm A (or B) and will therefore get an wage offer w for sure. In case of $(1,2,3)$ or $(1,3,2)$ worker 1 also gets an offer from a low-wage firm with certainty, since the workers that applied to the high-wage firm will accept the high-wage firm's offer (since $w^h > w$) and thus, there are two applicants at two low-wage firms which are both matched with certainty. Only in the $(0,3,3)$ case, worker 1 can fail to match (with probability 1/3), because 3 equal workers are competing for 2 jobs.

Payoff for a worker who sends both applications to the low-wage firms, i.e.,

H2	A2	B2	probability	H	A	B	π^{WH}
2	1	1	$\frac{1}{2}\xi^2$	2	2	2	w
2	2	0	$\frac{1}{4}\xi^2$	2	3	1	w
2	0	2	$\frac{1}{4}\xi^2$	2	1	3	w
1	1	2	$\xi(1-\xi)$	1	2	3	w
1	2	1	$\xi(1-\xi)$	1	3	2	w
0	2	2	$(1-\xi)^2$	0	3	3	$\frac{2}{3}w$

Table 11: Worker payoffs in the directed search with commitment case if the marginal worker sends both applications to low-wage firms

Equilibrium wages and application probabilities In equilibrium, all firms must make again equal profits regardless of the wage they post, i.e. $\pi^{FH} = \pi^{FL}$. Workers send their applications such that they maximize their expected payoff given that at most two different wages are offered by three firms. Workers are either indifferent between sending one of their applications to the high-wage firm and the other one to one of the low-wage firms (mixed strategy equilibrium), i.e. $\pi^{WH} = \pi^{WL}$, or they strictly prefer to send one application to the high-wage firm and send the second one to one of the low-wage firms (pure strategy equilibrium), i.e. $\pi^{WH} > \pi^{WL}$. Note, that sending both applications to both low-wage firms with certainty cannot be an equilibrium, since it would violate the equal profit condition. Given the workers' optimal application strategy firms must have no incentive to deviate from the wages offered in equilibrium.

Case 1: Mixed strategy equilibrium

If the high-wage firm offers a higher wage, i.e. $w^h > w$, the workers' indifference condition requires

$$\begin{aligned}
&\frac{1}{2}\xi^2\left(\frac{1}{3}w^h + \frac{2}{3}w\right) + \frac{1}{4}\xi^2\left(\frac{1}{3}w^h + \frac{1}{3}w\right) + \frac{1}{4}\xi^2\left(\frac{1}{3}w^h + \frac{2}{3}w\right) + \xi(1-\xi)\left(\frac{1}{2}w^h + \frac{5}{16}w\right) \\
&+\xi(1-\xi)\left(\frac{1}{2}w^h + \frac{3}{8}w\right) + (1-\xi)^2 w^h \\
=\ & \xi^2 w + 2\xi(1-\xi)w + (1-\xi)^2\frac{2}{3}w
\end{aligned}$$

Simplifying gives

$$\begin{aligned}
\frac{11}{16}w\xi + w^h - \xi w^h - \frac{5}{48}w\xi^2 + \frac{1}{3}\xi^2 w^h &= \frac{2}{3}w + \frac{2}{3}w\xi - \frac{1}{3}w\xi^2 \\
w^h - \xi w^h + \frac{1}{3}\xi^2 w^h &= \frac{2}{3}w - \frac{1}{48}w\xi - \frac{11}{48}w\xi^2 \\
\frac{w^h}{w} &= \frac{\frac{2}{3} - \frac{1}{48}\xi - \frac{11}{48}\xi^2}{1 - \xi + \frac{1}{3}\xi^2} \qquad (21)
\end{aligned}$$

Given that $w^h > w$, there exists an application probability $\xi > 0.46423$ such that workers are indifferent between applying to the high-wage firm and a low-wage firm.

Applying the implicit-function theorem to the workers' indifference condition gives a relation between the expected number of applications and the offered wages.

$$\left.\frac{d\xi}{dw^h}\right|_{w^h>w} = \frac{1 - \xi + \frac{1}{3}\xi^2}{\left(-\frac{1}{48} - \frac{22}{48}\xi\right)w - \left(-1 + \frac{2}{3}\xi\right)w^h} \qquad (22)$$

$$\left.\frac{d\xi}{dw}\right|_{w^h>w} = \frac{\frac{2}{3} - \frac{1}{48}\xi - \frac{11}{48}\xi^2}{\left(-1 + \frac{2}{3}\xi\right)w^h - \left(-\frac{1}{48} - \frac{22}{48}\xi\right)w} \qquad (23)$$

Firms choose the wage that maximizes profits,

$$\frac{\partial \pi^{FH}}{\partial w^h} = -\left(3\xi - 3\xi^2 + \xi^3\right) + \left(1 - w^h\right)\left(3 - 6\xi - 3\xi^2\right) \left.\frac{d\xi}{dw^h}\right|_{w^h > w} = 0 \tag{24}$$

$$\frac{\partial \pi^{FL}}{\partial w} = -\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right) - (1 - w)\left(\frac{3}{16}\xi^2 - \frac{10}{16}\xi\right) \left.\frac{d\xi}{dw}\right|_{w^h > w} = 0 \tag{25}$$

Rewriting the firm's equal profit condition, (20) yields,

$$w^h = 1 - \frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)}(1 - w)$$

Using (22) to eliminate $\frac{d\xi}{dw^h}$ in (24) and use the equation above to eliminate w^h in (24) gives,

$$\begin{aligned} 0 = & -\left(3\xi - 3\xi^2 + \xi^3\right) \\ & + \frac{(1 - w)\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)} \frac{\left(3 - 6\xi - 3\xi^2\right)\left(1 - \xi + \frac{1}{3}\xi^2\right)}{\left(-\frac{1}{48} - \frac{22}{48}\xi\right) w - \left(-1 + \frac{2}{3}\xi\right)\left(1 - \frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)(1 - w)}{\left(3\xi - 3\xi^2 + \xi^3\right)}\right)} \end{aligned}$$

$$w = \frac{\begin{array}{c} \frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)}\left(3 - 6\xi - 3\xi^2\right)\left(1 - \xi + \frac{1}{3}\xi^2\right) - \left(3\xi - 3\xi^2 + \xi^3\right) \\ \left(-1 + \frac{2}{3}\xi\right)\left(\frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)} - 1\right) \end{array}}{\begin{array}{c} \left(3\xi - 3\xi^2 + \xi^3\right)\left(-\frac{1}{48} - \frac{22}{48}\xi\right) - \left(\frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)} - 1\right)\left(3\xi - 3\xi^2 + \xi^3\right)\left(-1 + \frac{2}{3}\xi\right) \\ + \frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)}\left(3 - 6\xi - 3\xi^2\right)\left(1 - \xi + \frac{1}{3}\xi^2\right) \end{array}} \tag{26}$$

Using (23) to eliminate $\frac{d\xi}{dw}$ in (25) gives,

$$-\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right) - \frac{(1 - w)\left(\frac{3}{16}\xi^2 - \frac{10}{16}\xi\right)\left(\frac{2}{3} - \frac{1}{48}\xi - \frac{11}{48}\xi^2\right)}{\left(-1 + \frac{2}{3}\xi\right)\left(1 - \frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)}(1 - w)\right) - \left(-\frac{1}{48} - \frac{22}{48}\xi\right) w} = 0$$

Solving for w yields,

$$w = \frac{\begin{array}{c} \left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)\left(-1 + \frac{2}{3}\xi\right) - \left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)\left(-1 + \frac{2}{3}\xi\right)\frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)} \\ + \left(\frac{3}{16}\xi^2 - \frac{10}{16}\xi\right)\left(\frac{2}{3} - \frac{1}{48}\xi - \frac{11}{48}\xi^2\right) \end{array}}{\begin{array}{c} \left(\frac{3}{16}\xi^2 - \frac{10}{16}\xi\right)\left(\frac{2}{3} - \frac{1}{48}\xi - \frac{11}{48}\xi^2\right) - \left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)\left(-1 + \frac{2}{3}\xi\right)\frac{\left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)}{\left(3\xi - 3\xi^2 + \xi^3\right)} \\ + \left(\frac{1}{16}\xi^3 - \frac{5}{16}\xi^2 + 1\right)\left(-\frac{1}{48} - \frac{22}{48}\xi\right) \end{array}} \tag{27}$$

The equilibrium application probability follows from equating (26) and (27),

$$\frac{\frac{\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)}{\left(3\xi-3\xi^2+\xi^3\right)}\left(3-6\xi-3\xi^2\right)\left(1-\xi+\frac{1}{3}\xi^2\right) - \left(3\xi-3\xi^2+\xi^3\right)\left(-1+\frac{2}{3}\xi\right)\left(\frac{\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)}{\left(3\xi-3\xi^2+\xi^3\right)}-1\right)}{\left(3\xi-3\xi^2+\xi^3\right)\left(-\frac{1}{48}-\frac{22}{48}\xi\right)-\left(\frac{\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)}{\left(3\xi-3\xi^2+\xi^3\right)}-1\right)\left(3\xi-3\xi^2+\xi^3\right)\left(-1+\frac{2}{3}\xi\right) + \frac{\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)}{\left(3\xi-3\xi^2+\xi^3\right)}\left(3-6\xi-3\xi^2\right)\left(1-\xi+\frac{1}{3}\xi^2\right)}$$

$$= \frac{\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)\left(-1+\frac{2}{3}\xi\right)-\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)\left(-1+\frac{2}{3}\xi\right)\frac{\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)}{\left(3\xi-3\xi^2+\xi^3\right)} + \left(\frac{3}{16}\xi^2-\frac{10}{16}\xi\right)\left(\frac{2}{3}-\frac{1}{48}\xi-\frac{11}{48}\xi^2\right)}{\left(\frac{3}{16}\xi^2-\frac{10}{16}\xi\right)\left(\frac{2}{3}-\frac{1}{48}\xi-\frac{11}{48}\xi^2\right)-\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)\left(-1+\frac{2}{3}\xi\right)\frac{\left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)}{\left(3\xi-3\xi^2+\xi^3\right)} + \left(\frac{1}{16}\xi^3-\frac{5}{16}\xi^2+1\right)\left(-\frac{1}{48}-\frac{22}{48}\xi\right)}$$

or

$$\begin{aligned} & \left(258\xi^3-239\xi^2-48\xi-122\xi^4+27\xi^5+48\right)\begin{pmatrix} 560\xi+2448\xi^2-2749\xi^3+346\xi^4 \\ +782\xi^5-404\xi^6+57\xi^7-768 \end{pmatrix} \\ = & \left(195\xi^3-242\xi^2-48\xi-57\xi^4+5\xi^5+48\right)\begin{pmatrix} 2816\xi-2400\xi^2-110\xi^3+1138\xi^4 \\ -566\xi^5+75\xi^6+3\xi^7-768 \end{pmatrix} \end{aligned}$$

The equilibrium application probability is $\xi = 0.956$. Substituting this into (26) or (27) gives $w = 0.772$. The equal profit condition then implies $w^h = 0.825$.

Case 2: Non existence of a pure strategy equilibrium, i.e. $\xi = 1$

The equal profit condition with $\xi = 1$ requires

$$\frac{1-w^h}{1-w} = \frac{\frac{1}{16}-\frac{5}{16}+1}{3-3+1} = \frac{3}{4} \quad \text{or} \quad w^h = \frac{1}{4}+\frac{3}{4}w \quad \text{or} \quad w = \frac{4}{3}w^h - \frac{1}{3}$$

Using the first order condition of the high-wage firm and using (22) to eliminate $\frac{d\xi}{dw^h}$ implies at $\xi = 1$,

$$\begin{aligned} \left.\frac{\partial \pi^{FH}}{\partial w^h}\right|_{\xi=1} &= -(3-3+1)+\left(1-w^h\right)(3-6-3)\frac{1-1+\frac{1}{3}}{\left(-\frac{1}{48}-\frac{22}{48}\right)w-\left(-1+\frac{2}{3}\right)w^h} \\ &= 2\frac{1-w^h}{\frac{23}{48}w-\frac{1}{3}w^h}-1 = 2\frac{1-\left(\frac{1}{4}+\frac{3}{4}w\right)}{\frac{23}{48}w-\frac{1}{3}\left(\frac{1}{4}+\frac{3}{4}w\right)}-1 \\ &= \frac{3}{2}\frac{1-w}{\frac{11}{48}w-\frac{1}{12}}-1. \end{aligned}$$

Doing the same for the first order condition for the low-wage firms and using (23) to illuminate $\frac{d\xi}{dw}$ implies at $\xi = 1$,

$$\begin{aligned}\left.\frac{\partial \pi^{FA}}{\partial w}\right|_{\xi=1} &= -\left(\frac{1}{16}-\frac{5}{16}+1\right)-(1-w)\left(\frac{3}{16}-\frac{10}{16}\right)\frac{\frac{2}{3}-\frac{1}{48}-\frac{11}{48}}{\left(-1+\frac{2}{3}\right)w^h-\left(-\frac{1}{48}-\frac{22}{48}\right)w}\\ &= \frac{35}{192}\frac{1-w}{\frac{23}{48}w-\frac{1}{3}w^h}-\frac{3}{4}=\frac{35}{192}\frac{1-w}{\frac{23}{48}w-\frac{1}{3}\left(\frac{1}{4}+\frac{3}{4}w\right)}-\frac{3}{4}\\ &= \frac{35}{192}\frac{1-w}{\frac{11}{48}w-\frac{1}{12}}-\frac{3}{4}.\end{aligned}$$

Comparing the first order condition for the high-wage firm and the low-wage firm implies that there exists no equilibrium wage w that ensures that no firm has an incentive to deviate.

A.4.2 Non existence of an equilibrium with two high and one low-wage firm

In order to calculate the hiring probabilities, note that in the $(2, 2, 2)$ case the low-wage firm remains unmatched, if it competes with one of the high wage firms (A or B) in round one for the same worker, which occurs with probability $1/2 \times 1/2$ (the probability that the worker is first offered the job at both firms). In the second round firm L offers the job to the remaining worker. If this worker was offered the job at firm B (or A) in round one, which happens with probability 1/2, then the worker will turn down the low-wage firm's offer in round two, since $w^l < w$. So the hiring probability in this case is $1 - \frac{1}{2}\frac{1}{2}\frac{1}{2} = \frac{7}{8}$.

Next, note that if the low-wage firm received only one application, then it is not matched, if its worker receives the first offer from the firm with 3 applications, which happens with probability 1/3. If not, which occurs with probability 2/3, the firm with three applicants will offer the job to the same worker as the firm with 2 applicants with probability 1/2 (suppose worker 1 is at the low-wage firm, then worker 2 and 3 applied at firm A and B. Conditional on not picking worker 1, the probability that firms A and B pick the same worker is $1/2 = 2 \times 1/2 \times 1/2$). Since firms A and B offer the same wage, the firm with 3 applications will not get the first worker with probability 1/2. In this case, the firm with 3 applications offers in the second round with probability 1/2 the job to the same worker that applied at firm L. Since $w^l < w$, the low-wage firm will not be matched. So in this case, the hiring probability is, $1 - \left(\frac{1}{3} + \frac{2}{3}\frac{1}{2}\frac{1}{2}\frac{1}{2}\right) = \frac{7}{12}$. Table 12 below summarizes the matching probabilities and payoffs for the low-wage firm L.

Now consider the matching probability of the high-wage firms. If they have 3 applications, they are matched with certainty. If they got 2 applications, they also match with certainty, since they pay a higher wage than the low-wage firm. A high-wage firm with 2 applications also matches with certainty, if the other high-wage firm has 3 applicants, since none of the two common applicants can be hired by the low-wage firm given that $w^l < w$. The same argument applies if all three firms have two applicants. If a high-wage firm has one applicant and the low-wage firm 3 applicants, then the high-wage firm also hires with certainty since $w^l < w$. If one high-wage firm has one application and the other has 3 applications, then the high-wage firm with 1 application does not get matched with certainty. This happens if the same worker is first at the high-wage firm with 3 applications (which happens with probability 1/3) and accepts the offer of the firm with 3 applicants (which happens with probability 1/2). The matching probabilities are summarized in Table 12.

L	A	B	probabilities	π^{FL}	$\pi^{FH}(A)$	$\pi^{FH}(B)$
3	1	2	$\frac{3}{8}\xi^3$	1	1	1
3	2	1	$\frac{3}{8}\xi^3$	1	1	1
3	3	0	$\frac{1}{8}\xi^3$	1	1	0
3	0	3	$\frac{1}{8}\xi^3$	1	0	1
2	2	2	$\frac{2}{4}3\xi^2(1-\xi)$	$\frac{7}{8}$	1	1
2	3	1	$\frac{1}{4}3\xi^2(1-\xi)$	1	1	$\frac{5}{6}$
2	1	3	$\frac{1}{4}3\xi^2(1-\xi)$	1	$\frac{5}{6}$	1
1	3	2	$\frac{1}{2}3\xi(1-\xi)^2$	$\frac{7}{12}$	1	1
1	2	3	$\frac{1}{2}3\xi(1-\xi)^2$	$\frac{7}{12}$	1	1
0	3	3	$(1-\xi)^3$	0	1	1

Table 12: Firms' matching probabilities with two high- and one low-wage firm

The respective profits of the low- and high-wage firms follow from this table are given by

$$\begin{aligned}\pi^{FL} &= \left(1-w^l\right)\left(\xi^3+\frac{1}{2}3\xi^2(1-\xi)\frac{7}{8}+\frac{1}{2}3\xi^2(1-\xi)+3\xi(1-\xi)^2\frac{7}{12}\right)\\ &= \left(1-w^l\right)\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)\end{aligned}$$

$$\begin{aligned}\pi^{FH} &= (1-w)\left(\frac{7}{8}\xi^3+\frac{3}{4}3\xi^2(1-\xi)+\frac{1}{4}3\xi^2(1-\xi)\frac{5}{6}+3\xi(1-\xi)^2+(1-\xi)^3\right)\\ &= (1-w)\left(1-\frac{1}{8}\xi^2\right)\end{aligned}$$

$$\frac{\left(1-w^{l}\right)}{(1-w)}=\frac{1-\frac{1}{8}\xi^{2}}{\frac{7}{4}\xi-\frac{11}{16}\xi^{2}-\frac{1}{16}\xi^{3}}$$

Since $\frac{1-w^l}{1-w} > 1$, by assumption $w^l < w$, it follows that only an equilibrium in mixed strategies exists, since applying to the high-wage firm with certainty, i.e. $\xi = 1$, contradicts $\frac{1-w^l}{1-w} > 1$. In a mixed strategy equilibrium workers have to be indifferent between sending both applications to low-wage firms and sending one application to a high-wage firm and one to the low-wage firm. The equal profit condition implies that any $\xi < 0.789$ can be an equilibrium solution.

The first order conditions for the high and the low-wage firms that determine the wages at which firms have no incentive to deviate are given by

$$\begin{aligned}
\frac{\partial\pi^{FL}}{\partial w^{l}} &= -\left(\frac{7}{4}\xi-\frac{11}{16}\xi^{2}-\frac{1}{16}\xi^{3}\right)+\left(1-w^{l}\right)\left(\frac{7}{4}-\frac{22}{16}\xi-\frac{3}{16}\xi^{2}\right)\left.\frac{d\xi}{dw^{l}}\right|_{w^{l}<w}=0,\\
\frac{\partial\pi^{F}}{\partial w} &= -\left(1-\frac{1}{8}\xi^{2}\right)-(1-w)\,\frac{1}{4}\xi\left.\frac{d\xi}{dw}\right|_{w^{l}<w}=0.
\end{aligned}$$

Worker payoffs Table 11 shows in columns 1 to 3, the possible networks that can arise. Suppose that worker 1 sends one application to the low-wage firm and one application to one of the high-wage firms (A or to B). The resulting network is then given by columns 5 to 7.

In the $(2,1,1)$ case, worker 1 is with two other applicants at the low-wage firm and with one other applicant at a high-wage firm. Since the other applicant at the high-wage firm also applied to the low-wage firm, worker 1 is always matched. With probability 1/2, the high-wage firm offers the worker a wage $w > w^l$, which he accepts. Otherwise worker one is hired at the low-wage firm firm at w^l.

In the $(2,2,0)$ or $(2,0,2)$ case we have to distinguish between the outcomes $(3,3,0)$ and $(3,0,3)$ on the one hand and $(3,2,1)$ and $(3,1,2)$ on the other hand. Clearly, in case $(3,3,0)$ or $(3,0,3)$ worker one is picked with probability 1/3 by a high-wage firm. If not, which occurs with probability 2/3, then the worker is one of two remaining workers at the low wage, i.e., she is hired with probability 1/2. In case of $(3,2,1)$ and $(3,1,2)$ the worker is offered the wage w by the high-wage firm with certainty.

In the $(1,1,2)$ or $(1,2,1)$ cases we have to distinguish between the final outcomes $(2,2,2)$ on the one hand and $(2,3,1)$ or $(2,1,3)$ on the other hand. If the final network is given by $(2,2,2)$, worker 1 receives with probability 1/2 an offer from the high-wage firm that she accepts. If worker 1 is second at the high-wage firm, there is a chance that she still gets an offer from the high-wage firm. Namely, if the first worker at the high-wage firm is also the first worker at the other high-wage firm (which happens with probability 1/2) and if this worker accepts the offer at the other high-wage firm (which happens with probability 1/2). Thus, the probability that worker 1 gets an offer from the high-wage firm is given by $\frac{1}{2}+\frac{1}{2}\frac{1}{2}\frac{1}{2}$.

Next, we calculate the probability that worker 1 gets and accepts an offer from the low-wage firm. She gets an offer, if she is first (probability 1/2), or (probability 1/2) if the first worker at the low wage firm does not accept the offer from the low-wage firm, which happens only, if this worker is offered a job at a high-wage firm (which happens with probability $1/2+(1/2\times 1/2\times 1/2)$ as calculated above). Since $w^l<w$, worker 1 only accepts the offer from the low-wage firm, if she does not get an offer from a high-wage firm (which happens with probability $1-(1/2+1/2\times 1/2\times 1/2)$)

$$\frac{1}{2}w+\frac{1}{2}\frac{1}{2}\frac{1}{2}w+\left(1-\left(\frac{1}{2}+\frac{1}{2}\frac{1}{2}\frac{1}{2}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\frac{1}{2}\frac{1}{2}\right)\right)w^l=\frac{5}{8}w+\frac{39}{128}w^l$$

If the final network is given by $(2,3,1)$ or $(2,1,3)$ the worker receives with probability 1/3 an offer from a high-wage firm and accepts it. If worker 1 is not first at a high-wage firm, there is a chance that the worker still gets an offer from this high-wage firm. Namely, if the high-wage firm with three applications picks first the worker that also applied to the other high-wage firm (which happens with probability 1/3). In this case the selected worker will reject the offer at the high-wage firm with 3 applications with probability 1/2. If (in this case) the high-wage firm with 3 applications picks in the second round worker 1 (which happens with probability 1/2), then worker 1 will receive wage w. Thus, the probability that worker 1 is offered a wage by a high-wage firm is given by $\frac{1}{3}+\frac{1}{3}\frac{1}{2}\frac{1}{2}$.

Next, we calculate the probability that worker 1 receives an offer from the low-wage firm. She gets an offer, if she is first (probability 1/2), or if the first worker at the low-wage firm does not accept the offer from the low-wage firm, which happens only, if the this worker is offered a job at the high-wage firm (which happens with probability $1/3+1/3\times 1/2\times 1/2$ as calculated above). Since $w^l<w$, worker 1 only accepts the offer from the low-wage firm, if she does not get an offer from a high-wage firm (which happens with probability $1-(1/3+1/3\times 1/2\times 1/2)$). Adding up yields,

$$\frac{1}{3}w+\frac{1}{3}\frac{1}{2}\frac{1}{2}w+\left(1-\left(\frac{1}{3}+\frac{1}{3}\frac{1}{2}\frac{1}{2}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(\frac{1}{3}+\frac{1}{3}\frac{1}{2}\frac{1}{2}\right)\right)w^l=\frac{5}{12}w+\frac{119}{288}w^l$$

Finally, in the $(0,2,2)$ case, worker 1 will be one out of three applicants at the high-wage firm and the only applicant at the low-wage firm. The payoff is therefore $(1/3)\,w$ and $(2/3)\,w^l$. Table 13 summarizes the worker's payoffs,

L2	A2	B2	probability	L	A	B	probability	π^{WL}
2	1	1	$\frac{1}{2}\xi^2$	3	2	1	$\frac{1}{2}$	$\frac{1}{2}w + \frac{1}{2}w^l$
				3	1	2	$\frac{1}{2}$	$\frac{1}{2}w + \frac{1}{2}w^l$
2	2	0	$\frac{1}{4}\xi^2$	3	3	0	$\frac{1}{2}$	$\frac{1}{3}w + \frac{1}{3}w^l$
				3	2	1	$\frac{1}{2}$	w
2	0	2	$\frac{1}{4}\xi^2$	3	1	2	$\frac{1}{2}$	w
				3	0	3	$\frac{1}{2}$	$\frac{1}{3}w + \frac{1}{3}w^l$
1	1	2	$\xi(1-\xi)$	2	2	2	$\frac{1}{2}$	$\frac{5}{8}w + \frac{39}{128}w^l$
				2	1	3	$\frac{1}{2}$	$\frac{5}{12}w + \frac{119}{288}w^l$
1	2	1	$\xi(1-\xi)$	2	3	1	$\frac{1}{2}$	$\frac{5}{12}w + \frac{119}{288}w^l$
				2	2	2	$\frac{1}{2}$	$\frac{5}{8}w + \frac{39}{128}w^l$
0	2	2	$(1-\xi)^2$	1	3	2	$\frac{1}{2}$	$\frac{1}{3}w + \frac{2}{3}w^l$
				1	2	3	$\frac{1}{2}$	$\frac{1}{3}w + \frac{2}{3}w^l$

Table 13: Worker payoffs with two high and one low-wage firm for a worker who send one application to the low and one to a high wage firm

Table 14 presents the worker's payoffs, if she sends both applications to the high-wage firms.

If the final network is given by $(2, 2, 2)$ the worker gets no offer from any high-wage firm, if she is second at both firms (which happens with probability $1/2 \times 1/2$). In case of $(2, 3, 1)$ or $(2, 1, 3)$, worker 1 is the only worker at the high-wage firm A (or B) and will therefore get a wage offer w for sure. In case of $(1, 2, 3)$ or $(1, 3, 2)$ worker 1 is first at the high-wage firm with 3 applications with probability $1/3$ and gets an offer. If worker 1 is second (probability $1/3$), she either is first at the other high-wage firm with 2 applications (with probability $1/2$) and gets an offer there, or she is second at the other high-wage firm with 2 applications (with probability $1/2$) and the first worker at the high-wage firm with 2 applications is also the first worker at the high-wage firm with 3 applications (probability $1/2$). In this case worker 1 also gets matched. If worker 1 is third at the high-wage firm with 3 applications (probability $1/3$), she is first at the other high-wage firm with 2 applications (with probability $1/2$) and gets an offer there, or she is second at the other high-wage firm with 2 applications (with probability $1/2$) and the first worker at the high-wage firm with 2 applications is also the first worker at the high-wage firm with 3 applications (probability $1/2$) and the first worker accepts the offer at the high-wage firm with 3 applications with probability 1/2. So her payoff is,

$$\left(\frac{1}{3}+\frac{1}{3}\left(\frac{1}{2}+\frac{1}{2}\frac{1}{2}\right)+\frac{1}{3}\left(\frac{1}{2}+\frac{1}{2}\frac{1}{2}\frac{1}{2}\right)\right)w=\frac{19}{24}w$$

In the $(0,3,3)$ case, the expected payoff of worker 1 is $2/3w$.

L2	A2	B2	probability	L	A	B	π^{WH}
2	1	1	$\frac{1}{2}\xi^2$	2	2	2	$\frac{3}{4}w$
2	2	0	$\frac{1}{4}\xi^2$	2	3	1	w
2	0	2	$\frac{1}{4}\xi^2$	2	1	3	w
1	1	2	$\xi(1-\xi)$	1	2	3	$\frac{19}{24}w$
1	2	1	$\xi(1-\xi)$	1	3	2	$\frac{19}{24}w$
0	2	2	$(1-\xi)^2$	0	3	3	$\frac{2}{3}w$

Table 14: Worker payoffs with two high and one low-wage firm for a worker who sends both applications to an high wage firm

Wages The first order condition for the deviating and non deviating firms is given by

$$\begin{aligned}
\frac{\partial\pi^{FL}}{\partial w^l} &= -\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)+\left(1-w^l\right)\left(\frac{7}{4}-\frac{22}{16}\xi-\frac{3}{16}\xi^2\right)\left.\frac{d\xi}{dw^l}\right|_{w^l<w}=0\\
\frac{\partial\pi^{FH}}{\partial w} &= -\left(1-\frac{1}{8}\xi^2\right)-(1-w)\frac{1}{4}\xi\left.\frac{d\xi}{dw}\right|_{w^l<w}=0
\end{aligned}$$

If the low-wage firm offers a lower wage, i.e. $w^l<w$, the indifference condition is given by

$$\begin{aligned}
& \frac{1}{2}\xi^2\left(\frac{1}{2}w+\frac{1}{2}w^l\right)+\frac{1}{4}\xi^2\left(\frac{1}{3}w+\frac{1}{3}w^l\right)+\frac{1}{4}\xi^2w+\xi(1-\xi)\\
& \left(\frac{5}{8}w+\frac{39}{128}w^l\right)+\xi(1-\xi)\left(\frac{5}{12}w+\frac{119}{288}w^l\right)\\
& +(1-\xi)^2\left(\frac{1}{3}w+\frac{2}{3}w^l\right)\\
= & \frac{1}{2}\xi^2\frac{3}{4}w+\frac{1}{2}\xi^2w+\xi(1-\xi)\frac{19}{24}w+\xi(1-\xi)\frac{19}{24}w+(1-\xi)^2\frac{2}{3}w
\end{aligned}$$

Simplifying yields

$$\begin{aligned}
\frac{1}{3}w+\frac{3}{8}w\xi+\frac{2}{3}w^l-\frac{709}{1152}\xi w^l-\frac{1}{8}w\xi^2+\frac{325}{1152}\xi^2w^l &= \frac{2}{3}w+\frac{1}{4}w\xi-\frac{1}{24}w\xi^2\\
\frac{2}{3}w^l-\frac{709}{1152}\xi w^l+\frac{325}{1152}\xi^2w^l &= \frac{1}{3}w-\frac{1}{8}w\xi+\frac{1}{12}w\xi^2
\end{aligned}$$

$$\frac{w^l}{w} = \frac{\frac{1}{3} - \frac{1}{8}\xi + \frac{1}{12}\xi^2}{\frac{2}{3} - \frac{709}{1152}\xi + \frac{325}{1152}\xi^2}$$

The implicit function theorem implies

$$\left.\frac{d\xi}{dw^l}\right|_{w^l<w} = \frac{\frac{2}{3} - \frac{709}{1152}\xi + \frac{325}{1152}\xi^2}{\left(-\frac{1}{8} + \frac{1}{6}\xi\right) w - \left(-\frac{709}{1152} + \frac{650}{1152}\xi\right) w^l}$$

$$\left.\frac{d\xi}{dw}\right|_{w^l<w} = \frac{-\left(\frac{1}{3} - \frac{1}{8}\xi + \frac{1}{12}\xi^2\right)}{\left(-\frac{1}{8} + \frac{1}{6}\xi\right) w - \left(-\frac{709}{1152} + \frac{650}{1152}\xi\right) w^l}$$

Substitute this into the first order condition of firms implies for the low wage,

$$-\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right) + \frac{\left(1-w^l\right)\left(\frac{7}{4} - \frac{22}{16}\xi - \frac{3}{16}\xi^2\right)\left(\frac{2}{3} - \frac{709}{1152}\xi + \frac{325}{1152}\xi^2\right)}{\left(-\frac{1}{8} + \frac{1}{6}\xi\right) w - \left(-\frac{709}{1152} + \frac{650}{1152}\xi\right) w^l} = 0$$

and the high-wage,

$$-\left(1 - \frac{1}{8}\xi^2\right) + \frac{(1-w)\frac{1}{4}\xi\left(\frac{1}{3} - \frac{1}{8}\xi + \frac{1}{12}\xi^2\right)}{\left(-\frac{1}{8} + \frac{1}{6}\xi\right) w - \left(-\frac{709}{1152} + \frac{650}{1152}\xi\right) w^l} = 0$$

We can use the equal profit condition to eliminate w^l, i.e.,

$$w^l = 1 - \frac{\left(1 - \frac{1}{8}\xi^2\right)(1-w)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)}.$$

This gives for the low-wage firm,

$$-\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right) + \frac{\frac{\left(1-\frac{1}{8}\xi^2\right)(1-w)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)}\left(\frac{7}{4} - \frac{22}{16}\xi - \frac{3}{16}\xi^2\right)\left(\frac{2}{3} - \frac{709}{1152}\xi + \frac{325}{1152}\xi^2\right)}{\left(-\frac{1}{8} + \frac{1}{6}\xi\right) w - \left(-\frac{709}{1152} + \frac{650}{1152}\xi\right)\left(1 - \frac{\left(1-\frac{1}{8}\xi^2\right)(1-w)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)}\right)} = 0$$

$$w = \frac{\frac{\left(1-\frac{1}{8}\xi^2\right)\left(\frac{7}{4} - \frac{22}{16}\xi - \frac{3}{16}\xi^2\right)\left(\frac{2}{3} - \frac{709}{1152}\xi + \frac{325}{1152}\xi^2\right)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)^2} - \frac{\left(1-\frac{1}{8}\xi^2\right)\left(-\frac{709}{1152} + \frac{650}{1152}\xi\right)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)} + \left(-\frac{709}{1152} + \frac{650}{1152}\xi\right)}{\frac{\left(1-\frac{1}{8}\xi^2\right)\left(\frac{7}{4} - \frac{22}{16}\xi - \frac{3}{16}\xi^2\right)\left(\frac{2}{3} - \frac{709}{1152}\xi + \frac{325}{1152}\xi^2\right)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)^2} - \frac{\left(1-\frac{1}{8}\xi^2\right)\left(-\frac{709}{1152} + \frac{650}{1152}\xi\right)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)} + \left(-\frac{1}{8} + \frac{1}{6}\xi\right)}$$

and for the high-wage firm,

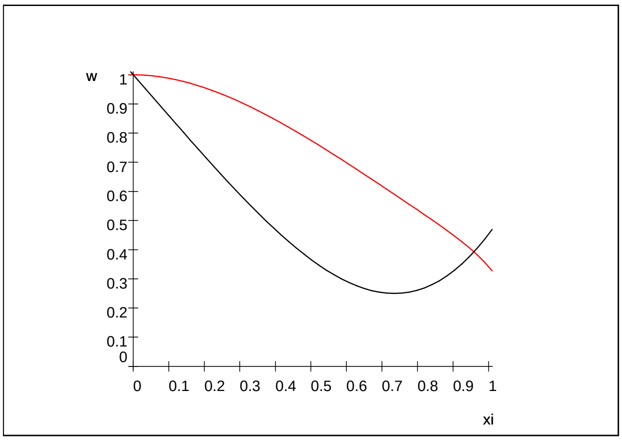
$$-\left(1 - \frac{1}{8}\xi^2\right) + \frac{(1-w)\frac{1}{4}\xi\left(\frac{1}{3} - \frac{1}{8}\xi + \frac{1}{12}\xi^2\right)}{\left(-\frac{1}{8} + \frac{1}{6}\xi\right) w - \left(-\frac{709}{1152} + \frac{650}{1152}\xi\right)\left(1 - \frac{\left(1-\frac{1}{8}\xi^2\right)(1-w)}{\left(\frac{7}{4}\xi - \frac{11}{16}\xi^2 - \frac{1}{16}\xi^3\right)}\right)} = 0$$

$$w=\frac{\frac{\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)\left(1-\frac{1}{8}\xi^2\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)}-\frac{\frac{1}{4}\xi\left(\frac{1}{3}-\frac{1}{8}\xi+\frac{1}{12}\xi^2\right)}{\left(1-\frac{1}{8}\xi^2\right)}-\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)}{\frac{\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)\left(1-\frac{1}{8}\xi^2\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)}-\frac{\frac{1}{4}\xi\left(\frac{1}{3}-\frac{1}{8}\xi+\frac{1}{12}\xi^2\right)}{\left(1-\frac{1}{8}\xi^2\right)}-\left(-\frac{1}{8}+\frac{1}{6}\xi\right)}$$

To get a value for ξ, we eliminate w from the above equations,

$$\frac{\frac{\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)\left(1-\frac{1}{8}\xi^2\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)}-\frac{\frac{1}{4}\xi\left(\frac{1}{3}-\frac{1}{8}\xi+\frac{1}{12}\xi^2\right)}{\left(1-\frac{1}{8}\xi^2\right)}-\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)}{\frac{\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)\left(1-\frac{1}{8}\xi^2\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)}-\frac{\frac{1}{4}\xi\left(\frac{1}{3}-\frac{1}{8}\xi+\frac{1}{12}\xi^2\right)}{\left(1-\frac{1}{8}\xi^2\right)}-\left(-\frac{1}{8}+\frac{1}{6}\xi\right)}$$

$$=\frac{\frac{\left(1-\frac{1}{8}\xi^2\right)\left(\frac{7}{4}-\frac{22}{16}\xi-\frac{3}{16}\xi^2\right)\left(\frac{2}{3}-\frac{709}{1152}\xi+\frac{325}{1152}\xi^2\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)^2}-\frac{\left(1-\frac{1}{8}\xi^2\right)\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)}+\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)}{\frac{\left(1-\frac{1}{8}\xi^2\right)\left(\frac{7}{4}-\frac{22}{16}\xi-\frac{3}{16}\xi^2\right)\left(\frac{2}{3}-\frac{709}{1152}\xi+\frac{325}{1152}\xi^2\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)^2}-\frac{\left(1-\frac{1}{8}\xi^2\right)\left(-\frac{709}{1152}+\frac{650}{1152}\xi\right)}{\left(\frac{7}{4}\xi-\frac{11}{16}\xi^2-\frac{1}{16}\xi^3\right)}+\left(-\frac{1}{8}+\frac{1}{6}\xi\right)}$$



The Figure above shows that the only solution that satisfies the equal profit condition, i.e. $\xi < 0.789$, is given by $\xi = 0$. Thus, no equilibrium $w^l < w$ exists.

B Proof of Proposition 1 (minimizing the fraction of firms with no workers)

Let μ be the probability that a vacancy receives no applicants, if workers fully randomize,

$$\mu = \left(1 - \frac{a}{v}\right)^u.$$

In a large labor market with $v, u \to \infty, \frac{v}{u} \to \theta$ (which we will assume from now on) this simplifies to

$$\mu = \exp\left(-\frac{a}{\theta}\right).$$

Next, suppose that some workers do not fully randomize over all vacancies. Suppose that qv vacancies are blue and that workers always send one of their applications to a blue vacancy and the other ones to one of the $(1-q)v$ remaining vacancies. First, consider $a = 2$. The total fraction of vacancies without applicants is then,

$$\mu_1 q + \mu_2 (1-q) = \left(q \exp\left(\frac{1}{q}\right) + (1-q)\exp\left(\frac{1}{(1-q)}\right)\right)\exp\left(-\frac{1}{\theta}\right).$$

Applying Jensen's inequality to the exponential function implies,

$$\left(q \exp\left(\frac{1}{q}\right) + (1-q)\exp\left(\frac{1}{(1-q)}\right)\right) > \exp(2),$$

so that the number of firms without any application is minimized at $q = \frac{1}{a} = \frac{1}{2}$. So for $a = 2$, the number of firms without candidates is smallest if workers apply to each firm with equal probability (i.e. when search is random and or there is no wage dispersion). The same statement is true for any number of applications $a > 2$, since Jensen's inequality implies,

$$\left(\sum_{i=1}^{a} q_i \exp\left(\frac{1}{q_i}\right)\right) > \exp(a),$$

where $\sum_{i=1}^{a} q_i = 1$.

C Simulation algorithm and decomposing a graph into subgraphs

In our simulations, we apply the following algorithm where step 2 follows Corominas-Bosch (2004) which is based on Hall's marriage theorem.

Step 1: Take a, u, v as given and let $a < v$. Generate a distribution of networks for 3 cases, (i) complete randomization; workers send their first application with probability $\frac{1}{v}$ to a particular firm and their next one with probability $\frac{1}{v-1}$ to the remaining $v-1$ firms, ... and their last one with probability $\frac{1}{v-a}$ to the remaining $v-a$ firms, (ii) partition the market in two groups of vacancies, A and B. Place a fraction q of the vacancies in group A and a fraction $(1-q)$ in group B and let each worker send one application to a vacancy in group A and the other $(a-1)$ applications to group B. Each firm in group A receives an application from worker 1 with probability $1/qv$ and the same for workers 2,...u. For the $a=3$ example, each firm in group B gets with probability $(a-1)/(1-q)v$ the second application of worker 1 and if it did not get the second one, it gets the third one with probability $(a-2)/((1-q)v-1)$ etc. The same holds for the other workers. For $q=\frac{1}{a}$, the arrival rate at each firm is the same and the only difference with (i) is that the market is partitioned.

Step 2: Determine the maximum number of matches on each network. As we showed before, ex post Bertrand competition is sufficient to realize this. The maximum matching can be found using the algorithm of Corominas-Bosch (2004) which we summarize below

Step 2a: Eliminate all vacancies that did not receive any applicants.

Step 2b: For $k=2,...v$, identify the groups of k vacancies that are jointly linked to less than k workers. Remove and collect them. We refer to those subgraphs as firm graphs.

Step 2c: Repeat step 2 but now reverse the role of workers and firms.

Step 2d: When all those subgraphs are removed, the remaining ones are balanced connected graphs (with an equal number of workers and firms)

Step 3: Index the firm graphs by f and the worker graphs by w and denote the total number of firm graphs by F, the total number of worker graphs by W and the number of even graphs by E, u_f is number of workers in firm graph f, v_w is number of firms in worker graph w. $f(i)$ is the number of firms in firm graph f, $w(j)$ is number of workers in worker graph w. The number of matches, M, is then given by,

$$M=\sum_{f=1}^{F}u_f+\sum_{w=1}^{W}v_w+\sum_{e=1}^{E}u_e,$$

the fraction of firms in firm graphs is

$$I/v=\sum_{f=1}^{F}f(i)$$

and the fraction of workers in worker graphs is

$$J/u=\sum_{w=1}^{W}w(j).$$